NCERT Solution For Class 10 Maths Chapter 2- Polynomials

Exercise 2.3 Page: 36

1. Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in each of the following:

(i) p(x) = $x^3 - 3x^2 + 5x - 3$, g(x) = $x^2 - 2$

Solution: Given,

Dividend = p(x) = $x^3 - 3x^2 + 5x - 3$

Divisor = g(x) = $x^2 - 2$

x - 3

$x^2 - 2)\ x^3 - 3x^2 + 5x - 3$

$x^3 \quad - 2x$

- +

$-3x^2 + 7x - 3$

$-3x^2 \quad - 3$

- +

7x – 9

Therefore, upon division we get,
Quotient = x – 3
Remainder = 7x – 9

(ii) p(x) = $x^4 - 3x^2 + 4x + 5$, g(x) = $x^2 + 1 - x$

Solution: Given,

Dividend = p(x) = $x^4 - 3x^2 + 4x + 5$

Divisor = g(x) = $x^2 + 1 - x$

$x^2 + x - 3$

$x^2 + 1 - x)\ x^4 - 3x^2 + 4x + 5$

$x^4 - x^3 + x^2$

$-\quad +\quad -$

$x^3 - 4x^2 + 4x + 5$

$x^3 - x^2 + x$

NCERT Solution For Class 10 Maths Chapter 2- Polynomials

$-$ + $-$

$-3x^2 + 3x + 5$

$-3x^2 + 3x - 5$
+ - +

8

Therefore, upon division we get,
Quotient = $x^2 + x - 3$
Remainder = 8

(iii) p(x) = $x^4 - 5x + 6$, g(x) = 2 – x^2

Solution: Given,

Dividend = p(x) =$x^4 - 5x + 6 = x^4 + 0x^2 - 5x + 6$

Divisor = g(x) = 2 – x^2 = – $x^2 + 2$

x - 3
$-x^2$ $+ 2)\ x^4 + 0x^2 - 5x + 6$
$x^4 - 2x^2$
$-$ +

$2x^2 - 5x + 6$
$2x^2 - 4$
$-$ +

-5x + 10

Therefore, upon division we get,
Quotient = x - 3
Remainder = -5x + 10

2. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(i) $t^2 - 3,\ 2t^4 + 3t^3 - 2t^2 - 9t - 12$

Solutions: Given,

NCERT Solution For Class 10 Maths Chapter 2- Polynomials

First polynomial = $t^2 - 3$

Second polynomial = $2t^4 + 3t^3 - 2t^2 - 9t - 12$

$$\begin{array}{r} 2t^2 + 3t + 4 \\ t^2 - 3 \,\big)\, 2t^4 + 3t^3 - 2t^2 - 9t - 12 \\ 2t^4 + 0t^3 - 6t^2 \\ - \quad\quad - \quad + \\ \hline 2t^3 + 4t^2 - 9t - 12 \\ 3t^3 + 0t^2 - 9t \\ - \quad\quad - \quad + \\ \hline 4t^2 - 0t - 12 \\ 4t^2 - 0t - 12 \\ - \quad\quad + \quad + \\ \hline 0 \\ \hline \end{array}$$

As we can see, the remainder is left as 0. Therefore, we say that, $t^2 - 3$ *is a factor of* $2t^2 + 3t + 4$.

(ii) $x^2 + 3x + 1, 3x^4 + 5x^3 - 7x^2 + 2x + 2$

Solutions: Given,

First polynomial = $x^2 + 3x + 1$

Second polynomial = $3x^4 + 5x^3 - 7x^2 + 2x + 2$

$$\begin{array}{r} 3x^2 + 4x + 2 \\ x^2 + 3x + 1 \,\big)\, 3x^4 + 5x^3 - 7x^2 + 2x + 2 \\ -(3x^4 + 5x^3 - 7x^2) \\ \hline -4x^3 - 10x^2 + 2x + 2 \\ - \quad (-4x^3 + 12x^2 - 4x) \\ \hline 2x^2 + 6x + 2 \\ -(2x^2 + 6x + 2) \\ \hline 0 \\ \hline \end{array}$$

As we can see, the remainder is left as 0. Therefore, we say that, $x^2 + 3x + 1$ *is a factor of* $3x^4 + 5x^3 - 7x^2 + 2x + 2$.

NCERT Solution For Class 10 Maths Chapter 2- Polynomials

(iii) $x^3 - 3x + 1$, $x^5 - 4x^3 + x^2 + 3x + 1$

Solutions: Given,

First polynomial = $x^3 - 3x + 1$
Second polynomial = $x^5 - 4x^3 + x^2 + 3x + 1$

```
                 x^2 - 1
               ________________________________
x^3 - 3x + 1)  x^5 - 4x^3 + x^2 + 3x + 1
               -(x^5 - 3x^3 + x^2)
               --------------------------------
                    -x^3 + 3x + 1
                    -(x^3 + 3x - 1)
               --------------------------------
2
               --------------------------------
```

As we can see, the remainder is not equal to 0. Therefore, we say that, $x^3 - 3x + 1$ *is not a factor of* $x^5 - 4x^3 + x^2 + 3x + 1$.

3. Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Solutions: Since this is a polynomial equation of degree 4, hence there will be total 4 roots.

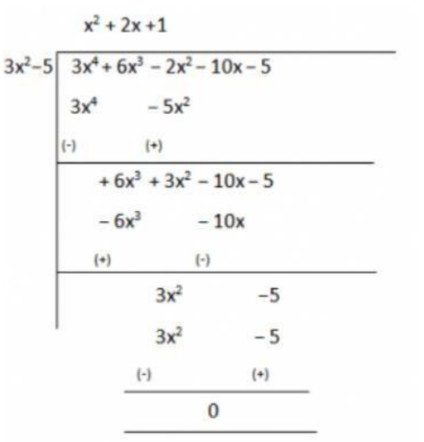
$\sqrt{\frac{5}{3}}$ *and* $-\sqrt{\frac{5}{3}}$ are zeroes of polynomial f(x).

∴ $(x - \sqrt{\frac{5}{3}})(x + \sqrt{\frac{5}{3}}) = x^2 - \frac{5}{3} = 0$

$(3x^2-5)=0$, is a factor of given polynomial f(x).

Now, when we will divide f(x) by $(3x^2-5)$ the quotient obtained will also be a factor of f(x) and the remainder

NCERT Solution For Class 10 Maths Chapter 2- Polynomials



will be 0.

Therefore, $3x^4 + 6x^3 - 2x^2 - 10x - 5 = (3x^2 - 5)(x^2 + 2x + 1)$

Now, on further factorizing $(x^2 + 2x + 1)$ we get,

$x^2 + 2x + 1 = x^2 + x + x + 1 = 0$

$x(x + 1) + 1(x+1) = 0$

$(x+1)(x+1) = 0$

So, its zeroes are given by: $x = -1$ and $x = -1$.

Therefore, all four zeroes of given polynomial equation are:

$\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}$, −1 and −1.

Hence, is the answer.

4. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial g(x), the quotient and remainder were x – 2 and –2x + 4, respectively. Find g(x).

Solutions: Given,
Dividend, p(x) = $x^3 - 3x^2 + x + 2$
Quotient = x-2
Remainder = –2x + 4
We have to find the value of Divisor, g(x) =?

As we know,
Dividend = Divisor × Quotient + Remainder
$\therefore x^3 - 3x^2 + x + 2 = g(x) \times (x-2) + (-2x + 4)$

NCERT Solution For Class 10 Maths Chapter 2- Polynomials

$x^3 - 3x^2 + x + 2 - (-2x + 4) = g(x) \times (x-2)$
Therefore, $g(x) \times (x-2) = x^3 - 3x^2 + 3x - 2$

Now, for finding g(x) we will divide $x^3 - 3x^2 + 3x - 2$ with (x-2)

Quotient: $x^2 - x + 1$

Divisor: $x - 2$; Dividend: $x^3 - 3x^2 + 3x - 2$

$x^3 - 2x^2$
(-) (+)

$-x^2 + 3x - 2$
$-x^2 + 2x$
(+) (-)

$x - 2$
$x - 2$
(-) (+)

0

Therefore, **g(x) = (x^2 – x +1)**

5. Give examples of polynomials p(x), g(x), q(x) and r(x), which satisfy the division algorithm and (i) deg p(x) = deg q(x)
(ii) deg q(x) = deg r(x)
(iii) deg r(x) = 0

Solutions: According to the division algorithm, dividend p(x) and divisor g(x) are two polynomials, where g(x)≠0. Then we can find the value of quotient q(x) and remainder r(x), with the help of below given formula;

Dividend = Divisor × Quotient + Remainder
∴ p(x) = g(x) × q(x) + r(x)
Where r(x) = 0 or degree of r(x)< degree of g(x).
Now let us proof the three given cases as per division algorithm by taking examples for each.

(i): deg p(x) = deg q(x)
Degree of dividend is equal to degree of quotient, only when the divisor is a constant term.
Let us take an example, $3x^2 + 3x + 3$ is a polynomial to be divided by 3.
So, $3x^2 + 3x + 3 \div 3 = x^2 + x + 1$ = q(x)
Thus, you can see, the degree of quotient is equal to the degree of dividend. Hence, division algorithm is satisfied here.

(ii): deg q(x) = deg r(x)
Let us take an example,p(x)= $x^2 + x$ is a polynomial to be divided by g(x)=x.
So, $x^2 + x \div x$ = x = q(x)

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Also, remainder, r(x) = x
Thus, you can see, the degree of quotient is equal to the degree of remainder. Hence, division algorithm is satisfied here.

(iii): deg r(x) = 0
The degree of remainder is 0 only when the remainder left after division algorithm is constant.
Let us take an example, p(x)= $x^2 + 1$ is a polynomial to be divided by g(x)=x.
So, $x^2 + 1 \div x$ = x = q(x)
And r(x)=1
Clearly, the degree of remainder here is 0.
Hence, division algorithm is satisfied here.

NCERT Solution For Class 10 Maths Chapter 2- Polynomials

Exercise 2.4 Page: 36

1. Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

(i) $2x^3 + x^2 - 5x + 2; \frac{1}{2}, 1, -2$

Solutions: Given, p(x) = $2x^3 + x^2 - 5x + 2$

And zeroes for p(x) are = $\frac{1}{2}, 1, -2$

$\therefore$ p(1/2)= $2(\frac{1}{2})^3 + (\frac{1}{2})^2 -$ 5(1/2) + 2 = ¼ + ¼ - 5/2 +2 = 0

p(1)= 2. $1^3 + 1^2 - 5.1 + 2 = 0$

p(-2)= $2(-2)^3 + (-2)^2 - 5(-2) + 2 = 0$

Hence, proved $\frac{1}{2}, 1, -2$ are the zeroes of $2x^3 + x^2 - 5x + 2$.

Now, comparing the given polynomial with general expression, we get;

$\therefore ax^3 + bx^2 + cx + d = 2x^3 + x^2 - 5x + 2$
a=3, b=1, c= -5 and d = 2

As we know, if α, β, γ are the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ then;

α + β + γ = –b/a

αβ + βγ + γα = c/a

α β γ = – d/a.

Therefore, putting the values of zeroes of the polynomial,

α + β + γ = ½+1+(-2) = -1/2 = –b/a

NCERT Solution For Class 10 Maths Chapter 2- Polynomials

αβ + βγ + γα = (1/2 ×1) + (1 ×-2) +(-2 ×1/2) = -5/2 = c/a

α β γ = ½ × 1 × (-2) = -2/2 = -d/a
Hence, the relationship between the zeroes and the coefficients are satisfied.

(ii) $x^3 - 4x^2 + 5x + 2$; 2, 1, 1

Solutions: Given, p(x) = $x^3 - 4x^2 + 5x + 2$
And zeroes for p(x) are 2, 1, 1.

∴ p(2)= $2^3 - 4.2^2 + 5.2 + 2 = 0$

p(1) = $1^3 - 4.1^2 + 5.1 + 2 = 0$
Hence proved, 2, 1, 1 are the zeroes of $x^3 - 4x^2 + 5x + 2$.

Now, comparing the given polynomial with general expression, we get;

∴ $ax^3 + bx^2 + cx + d$=$\mathbf{x^3 - 4x^2 + 5x + 2}$

a=1, b = -4, c = 5 and d = 2

As we know, if α, β, γ are the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ then;

α + β + γ = –b/a αβ
+ βγ + γα = c/a α β
γ = – d/a.

Therefore, putting the values of zeroes of the polynomial,

α + β + γ = 2+1+1 = 4 = -(-4)/1 = –b/a

αβ + βγ + γα = 2.1+1.1+1.2 = 5 = 5/1= c/a

α β γ = 2 × 1 × 1 = 2 = -(-2)/1 = -d/a
Hence, the relationship between the zeroes and the coefficients are satisfied.

2. Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, –7, –14 respectively.

Solutions: Let us consider the cubic polynomial is $ax^3 + bx^2 + cx + d$ and the values of the zeroes of the polynomials be α, β, γ.

As per the given question,

NCERT Solution For Class 10 Maths Chapter 2- Polynomials

α + β + γ = -b/a = 2/1

αβ + βγ + γα = c/a = -7/1

α β γ = -d/a = -14/1

Thus, from above three expressions we get the values of coefficient of polynomial.
a = 1, b = -2, c = -7, d = 14

Hence, the cubic polynomial is $x^3 - 2x^2 - 7x + 14$.

3. If the zeroes of the polynomial $x^3 - 3x^2 + x + 1$ are a – b, a, a + b, find a and b.

Solutions: We are given with the polynomial here,
p(x) = $x^3 - 3x^2 + x + 1$

And zeroes are given as a – b, a, a + b

Now, comparing the given polynomial with general expression, we get;

$\therefore px^3 + qx^2 + rx + s = x^3 - 3x^2 + x + 1$

p = 1, q = -3, r = 1 and s = 1

Sum of zeroes = a – b + a + a + b

-q/p = 3a

Putting the values q and p.

-(-3)/1 = 3a

a=1

Thus, the zeroes are 1-b, 1, 1+b.

Now, product of zeroes = 1(1-b)(1+b)

-s/p=1-b^2

- b^2 1/1=1-

b $+ 1$ $^2 = 1$ =2

b=$\sqrt{2}$

Hence, 1-$\sqrt{2}$, 1, 1+$\sqrt{2}$ *are the zeroes of* $x^3 - 3x^2 + x + 1$.

4. If two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are 2± $\sqrt{3}$, find other zeroes.

Solutions: Since this is a polynomial equation of degree 4, hence there will be total 4 roots.

Let f(x) = $x^4 - 6x^3 - 26x^2 + 138x - 35$

Since 2 + $\sqrt{3}$ and 2 -$\sqrt{3}$ are zeroes of given polynomial f(x).

$\therefore [x-(2+\sqrt{3})]\,[x-2-\sqrt{3}] = 0$

$(x-2-\sqrt{3})(x-2+\sqrt{3}) = 0$

On multiplying the above equation we get,

$x^2 - 4x + 1$, this is a factor of a given polynomial f(x).

Now, if we will divide f(x) by g(x), the quotient will also be a factor of f(x) and the remainder will be 0.

$$
\begin{array}{r|l}
 & x^2 - 2x - 35 \\
\hline
x^2 - 4x + 1 & x^4 - 6x^3 - 26x^2 + 138x - 35 \\
 & x^4 - 4x^3 + x^2 \\
 & (-)\ (+)\ (-) \\
\hline
 & -2x^3 - 27x^2 + 138x - 35 \\
 & -2x^3 + 8x^2 - 2x \\
 & (+)\ (-)\ (+) \\
\hline
 & -35x^2 + 140x - 35 \\
 & -35x^2 + 140x - 35 \\
 & (+)\ (-)\ (+) \\
\hline
 & 0
\end{array}
$$

So, $x^4 - 6x^3 - 26x^2 + 138x - 35 = (x^2 - 4x + 1)(x^2 - 2x - 35)$

Now, on further factorizing $(x^2 - 2x - 35)$ we get,

NCERT Solution For Class 10 Maths Chapter 2- Polynomials

x^2 – (7−5)x − 35 = x^2 – 7x + 5x +35 = 0 x(x

− 7) + 5 (x−7) = 0

(x+5) (x−7) = 0

So, its zeroes are given by:

x= −5 and x = 7.

Therefore, all four zeroes of given polynomial equation are: $2 + \sqrt{3}$, $2 - \sqrt{3}$, −5 and 7.