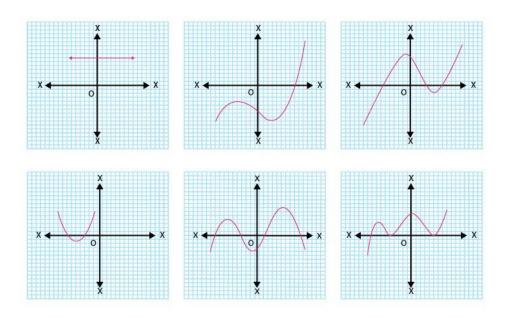


Exercise 2.1 Page: 28

1. The graphs of y = p(x) are given in Fig. 2.10 below, for some polynomials p(x). Find the number of zeroes of p(x), in each case.



Solutions:

Graphical method to find zeroes:-

Total number of zeroes in any polynomial equation = total number of times the curve intersects x-axis.

- (i) In the given graph, the number of zeroes of p(x) is 0 because the graph is parallel to x-axis does not cut it at any point.
- (ii) In the given graph, the number of zeroes of p(x) is 1 because the graph intersects the x-axis at only one point.
- (iii) In the given graph, the number of zeroes of p(x) is 3 because the graph intersects the x-axis at any three points.
- (iv) In the given graph, the number of zeroes of p(x) is 2 because the graph intersects the x-axis at two points.
- (v) In the given graph, the number of zeroes of p(x) is 4 because the graph intersects the x-axis at four points.
- (vi) In the given graph, the number of zeroes of p(x) is 3 because the graph intersects the x-axis at three points.



Exercise 2.2

Page: 33

1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

Solutions:

(i)
$$x^2$$
-2x -8
 $\Rightarrow x^2$ -4x+2x-8 = x(x-4)+2(x-4) = (x-4)(x+2)

Therefore, zeroes of polynomial equation x^2-2x-8 are (4, -2)

Sum of zeroes = $4-2=2=-(-2)/1=-(Coefficient of x)/(Coefficient of x^2)$

Product of zeroes = $4 \times (-2) = -8 = -(8)/1 = (Constant term)/(Coefficient of x^2)$

$(ii)4s^2-4s+1$

$$\Rightarrow$$
4s²-2s-2s+1 = 2s(2s-1)-1(2s-1) = (2s-1)(2s-1)

Therefore, zeroes of polynomial equation $4s^2-4s+1$ are (1/2, 1/2)

Sum of zeroes = $(\frac{1}{2})+(\frac{1}{2}) = 1 = -\frac{4}{4} = -(\text{Coefficient of s})/(\text{Coefficient of s}^2)$

Product of zeros = $(1/2)\times(1/2) = 1/4 = (Constant term)/(Coefficient of s^2)$

(iii) $6x^2-3-7x$

$$\Rightarrow$$
6x²-7x-3 = (3x+1)(2x-3)

Therefore, zeroes of polynomial equation $6x^2-3-7x$ are (-1/3, 3/2)

Sum of zeroes = $-(1/3)+(3/2) = (7/6) = -(Coefficient of x)/(Coefficient of x^2)$



Product of zeroes = $-(1/3)\times(3/2) = -(3/6) = (Constant term)/(Coefficient of x^2)$

$(iv)4u^2+8u$

 $\Rightarrow 4u(u+2)$

Therefore, zeroes of polynomial equation $4u^2 + 8u$ are (0, -2).

Sum of zeroes = $0+(-2) = -2 = -(8/4) = = -(Coefficient of u)/(Coefficient of u^2)$

Product of zeroes = $0 \times -2 = 0 = 0/4 = (Constant term)/(Coefficient of u^2)$

(v) t^2-15

 \Rightarrow t²= 15 or t = $\pm\sqrt{15}$ Thereforezeroes of polynomial equation t² -15 are ($\sqrt{15}$, - $\sqrt{15}$)

Sum of zeroes = $\sqrt{15+(-\sqrt{15})} = 0 = -(0/1) = -(Coefficient of t) / (Coefficient of t^2)$

Product of zeroes = $\sqrt{15} \times (-\sqrt{15}) = -15 = -15/1 = (Constant term) / (Coefficient of t^2)$

$(vi) 3x^2 - x - 4$

$$\Rightarrow 3x^2 - 4x + 3x - 4 = x(3x - 4) + 1(3x - 4) = (3x - 4)(x + 1)$$

Therefore, zeroes of polynomial equation $3x^2 - x - 4$ are (4/3, -1)

Sum of zeroes = $(4/3)+(-1) = (1/3) = -(-1/3) = -(Coefficient of x) / (Coefficient of x^2)$

Product of zeroes= $(4/3)\times(-1) = (-4/3) = (Constant term) / (Coefficient of x²)$

2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.



(i) 1/4, -1

Solution:

From the formulas of sum and product of zeroes, we know,

Sum of zeroes = $\alpha + \beta$

Product of zeroes = $\alpha \beta$

Sum of zeroes = $\alpha + \beta = 1/4$

Product of zeroes = $\alpha \beta$ = -1

.. If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:- x^2 -(α + β)x + α β = 0

$$x^2-(1/4)x+(-1)=0$$

$$4x^2-x-4=0$$

Thus, $4x^2-x-4$ is the quadratic polynomial.

(ii)√2, 1/3

Solution:

Sum of zeroes = $\alpha + \beta = \sqrt{2}$ Product of zeroes = $\alpha \beta = 1/3$

 \therefore If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$$x^2-(\alpha+\beta)x+\alpha\beta=0$$

$$x^2 - (\sqrt{2})x + (1/3) = 0$$

$$3x^2-3\sqrt{2}x+1=0$$

Thus, $3x^2-3\sqrt{2x+1}$ is the quadratic polynomial.

(iii) $0, \sqrt{5}$

Solution:



Given,

Sum of zeroes = $\alpha + \beta = 0$

Product of zeroes = $\alpha \beta = \sqrt{5}$

 \therefore If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$$x^2-(\alpha+\beta)x+\alpha\beta=0$$

$$x^2-(0)x+\sqrt{5}=0$$

Thus, $x^2 + \sqrt{5}$ is the quadratic polynomial.

(iv)1, 1

Solution:

Given,

Sum of zeroes = $\alpha + \beta = 1$

Product of zeroes = $\alpha \beta = 1$

∴ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:- x^2 -(α + β)x + α β = 0

$$x^2 - x + 1 = 0$$

Thus, x^2-x+1 is the quadratic polynomial.

(v) -1/4, 1/4

Solution:

Given,

Sum of zeroes = $\alpha + \beta = -1/4$

Product of zeroes = $\alpha \beta = 1/4$

 \therefore If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$$x^2-(\alpha+\beta)x+\alpha\beta=0$$



$$x^2-(-1/4)x+(1/4)=0$$

$$4x^2+x+1=0$$

Thus, $4x^2+x+1$ is the quadratic polynomial.

(vi) 4, 1

Solution:

Given,

Sum of zeroes = $\alpha + \beta = 4$

Product of zeroes = $\alpha\beta$ = 1

 \therefore If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$$x^2-(\alpha+\beta)x+\alpha\beta=0$$

$$x^2-4x+1=0$$

Thus, x^2 –4x+1 is the quadratic polynomial.



Exercise 2.3 Page: 36

1. Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in each of the following:

(i)
$$p(x) = x^3-3x^2+5x-3$$
, $g(x) = x^2-2$

Solution: Given,

Dividend =
$$p(x) = x^3 - 3x^2 + 5x - 3$$

Divisor =
$$g(x) = x^2 - 2$$

Therefore, upon division we get,

Quotient = x-3

Remainder = 7x-9

(ii)
$$p(x) = x^4-3x^2+4x+5$$
, $g(x) = x^2+1-x$

Solution: Given,

Dividend =
$$p(x) = x^4 - 3x^2 + 4x + 5$$

Divisor =
$$g(x) = x^2 + 1 - x$$



Therefore, upon division we get,

Quotient = $x^2 + x - 3$

Remainder = 8

(iii)
$$p(x) = x^4 - 5x + 6$$
, $g(x) = 2 - x^2$
Solution: Given,

Dividend =
$$p(x) = x^4 - 5x + 6 = x^4 + 0x^2 - 5x + 6$$

Divisor = $g(x) = 2 - x^2 = -x^2 + 2$

Therefore, upon division we get,



Quotient = $-x^2-2$ Remainder = -5x + 10

2. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(i)
$$t^2$$
-3, $2t^4$ +3 t^3 -2 t^2 -9 t -12 **Solutions:** Given,

First polynomial $= t^2-3$

Second polynomial = $2t^4 + 3t^3 - 2t^2 - 9t - 12$

As we can see, the remainder is left as 0. Therefore, we say that, t^2 -3 is a factor of $2t^2+3t+4$.

$$(ii)x^2+3x+1$$
, $3x^4+5x^3-7x^2+2x+2$

Solutions: Given,

First polynomial $= x^2 + 3x + 1$

Second polynomial = $3x^4+5x^3-7x^2+2x+2$



As we can see, the remainder is left as 0. Therefore, we say that, $x^2 + 3x + 1$ is a factor of $3x^4 + 5x^3 - 7x^2 + 2x + 2$.

(iii)
$$x^3-3x+1$$
, $x^5-4x^3+x^2+3x+1$

Solutions: Given,

First polynomial = x^3-3x+1

Second polynomial = $x^5-4x^3+x^2+3x+1$

As we can see, the remainder is not equal to 0. Therefore, we say that, x^3-3x+1 is not a factor of $x^5-4x^3+x^2+3x+1$.



3. Obtain all other zeroes of $3x^4+6x^3-2x^2-10x-5$, if two of its zeroes are $\sqrt{(5/3)}$ and $-\sqrt{(5/3)}$.

Solutions: Since this is a polynomial equation of degree 4, hence there will be total 4 roots.

 $\sqrt{(5/3)}$ and $-\sqrt{(5/3)}$ are zeroes of polynomial f(x).

$$(x - \sqrt{(5/3)}) (x + \sqrt{(5/3)}) = x^2 - (5/3) = 0$$

 $(3x^2-5)=0$, is a factor of given polynomial f(x).

Now, when we will divide f(x) by $(3x^2-5)$ the quotient obtained will also be a factor of f(x) and the remainder will be 0.

$$x^{2} + 2x + 1$$

$$3x^{2} - 5 \overline{\smash)3x^{4} + 6x^{3} - 2x^{2} - 10x - 5}$$

$$3x^{4} - 5x^{2}$$

$$(\cdot) \qquad (+)$$

$$+ 6x^{3} + 3x^{2} - 10x - 5$$

$$- 6x^{3} - 10x$$

$$(+) \qquad (\cdot)$$

$$3x^{2} - 5$$

$$3x^{2} - 5$$

$$(-) \qquad (+)$$

$$0$$

Therefore, $3x^4 + 6x^3 - 2x^2 - 10x - 5 = (3x^2 - 5)(x^2 + 2x + 1)$

Now, on further factorizing (x^2+2x+1) we get,

$$x^2+2x+1 = x^2+x+x+1 = 0$$

$$x(x+1)+1(x+1) = 0$$

$$(x+1)(x+1) = 0$$



So, its zeroes are given by: x = -1 and x = -1.

Therefore, all four zeroes of given polynomial equation are:

$$\sqrt{(5/3)}$$
, $\sqrt{(5/3)}$, -1 and -1 .

Hence, is the answer.

4. On dividing x^3-3x^2+x+2 by a polynomial g(x), the quotient and remainder were x-2 and -2x+4, respectively. Find g(x).

Solutions: Given,

Dividend, $p(x) = x^3 - 3x^2 + x + 2$

Quotient = x-2

Remainder = -2x+4

We have to find the value of Divisor, g(x) = ?

As we know,

Dividend = Divisor×Quotient + Remainder

$$x^3-3x^2+x+2 = g(x)\times(x-2)+(-2x+4)$$

$$x^3-3x^2+x+2-(-2x+4) = g(x)\times(x-2)$$

Therefore, $g(x) \times (x-2) = x^3-3x^2+x+2$

Now, for finding g(x) we will divide x^3-3x^2+x+2 with (x-2)

Therefore, $g(x) = (x^2-x+1)$



- 5. Give examples of polynomials p(x), g(x), q(x) and r(x), which satisfy the division algorithm and (i) deg p(x) = deg q(x)
- (ii) $\deg q(x) = \deg r(x)$
- (iii) deg r(x) = 0

Solutions: According to the division algorithm, dividend p(x) and divisor g(x) are two polynomials, where $g(x)\neq 0$. Then we can find the value of quotient q(x) and remainder r(x), with the help of below given formula;

Dividend = Divisor×Quotient+Remainder

$$p(x) = g(x) \times q(x) + r(x)$$

Where r(x) = 0 or degree of r(x) < degree of <math>g(x).

Now let us proof the three given cases as per division algorithm by taking examples for each.

(i):deg p(x) = deg q(x)

Degree of dividend is equal to degree of quotient, only when the divisor is a constant term.

Let us take an example, $3x^2+3x+3$ is a polynomial to be divided by 3.

So,
$$(3x^2+3x+3)/3 = x^2+x+1 = q(x)$$

Thus, you can see, the degree of quotient is equal to the degree of dividend. Hence, division algorithm is satisfied here.

(ii):deg q(x) = deg r(x)

Let us take an example , $p(x)=x^2+x$ is a polynomial to be divided by g(x)=x. So,

$$(x^2+x)/x = x+1 = q(x)$$

Also, remainder, r(x) = 0

Thus, you can see, the degree of quotient is equal to the degree of remainder. Hence, division algorithm is satisfied here.

(iii):deg r(x) = 0

The degree of remainder is 0 only when the remainder left after division algorithm is constant. Let us take an example, $p(x) = x^2 + 1$ is a polynomial to be divided by g(x) = x.

So,
$$(x^2+1)/x = x = q(x)$$

And r(x)=1

Clearly, the degree of remainder here is 0.

Hence, division algorithm is satisfied here.

Exercise 2.4

Page: 36

- 1. Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:
- (i) $2x^3+x^2-5x+2$; -1/2, 1, -2

Solutions: Given, $p(x) = 2x^3 + x^2 - 5x + 2$ And

zeroes for p(x) are = 1/2, 1, -2



$$p(1/2) = 2(1/2)^3 + (1/2)^2 - 5(1/2) + 2 = (1/4) + (1/4) - (5/2) + 2 = 0$$

$$p(1) = 2(1)^3 + (1)^2 - 5(1) + 2 = 0 \text{ p(-2)}$$

$$= 2(-2)^3 + (-2)^2 - 5(-2) + 2 = 0$$

Hence, proved 1/2, 1, -2 are the zeroes of $2x^3+x^2-5x+2$.

Now, comparing the given polynomial with general expression, we get;

$$ax^3+bx^2+cx+d=2x^3+x^2-5x+2$$

$$a=2$$
, $b=1$, $c=-5$ and $d=2$

As we know, if α , β , γ are the zeroes of the cubic polynomial ax $^3+bx^2+cx+d$, then;

$$\alpha + \beta + \gamma = -b/a$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = c/a$$
 α

$$\beta \gamma = -d/a$$
.

Therefore, putting the values of zeroes of the polynomial,

$$\alpha + \beta + \gamma = \frac{1}{2} + 1 + (-2) = -\frac{1}{2} = -\frac{b}{a} \alpha \beta + \beta \gamma + \gamma \alpha = (\frac{1}{2})$$

$$\times 1$$
)+ (1×-2) + $(-2 \times 1/2)$ = $-5/2$ = $c/a_{\alpha} \beta \gamma = \frac{1}{2}$

$$\times 1 \times (-2) = -2/2 = -d/a$$

Hence, the relationship between the zeroes and the coefficients are satisfied.

$(ii)x^3-4x^2+5x-2;2,1,1$

Solutions: Given, $p(x) = x^3-4x^2+5x-2$

$$p(2) = 2^3 - 4(2)^2 + 5(2) - 2 = 0$$

13-
$$4 \times 1^2 + 5 \times 1$$
 -2 = 0 And zeroes for p(x) are 2,1,1.
p(1) = () ()
Hence proved, 2, 1, 1 are the zeroes of x^3-4x^2+5x-2

Now, comparing the given polynomial with general expression, we get;

$$\therefore ax^3+bx^2+cx+d=x^3-4x^2+5x-2$$

$$a = 1$$
, $b = -4$, $c = 5$ and $d = -2$



As we know, if α , β , γ are the zeroes of the cubic polynomial ax $^3+bx^2+cx+d$, then;

$$\alpha + \beta + \gamma = -b/a \quad \alpha\beta$$

+ $\beta\gamma + \gamma\alpha = c/a \quad \alpha\beta$
 $\gamma = -d/a$.

Therefore, putting the values of zeroes of the polynomial,

$$\alpha + \beta + \gamma = 2 + 1 + 1 = 4 = -(-4)/1 = -b/a$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = 2 \times 1 + 1 \times 1 + 1 \times 2 = 5 = 5/1 = c/a$$

$$\alpha \beta \gamma = 2 \times 1 \times 1 = 2 = -(-2)/1 = -d/a$$

Hence, therelationship between the zeroes and the coefficients are satisfied.

2. Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, -7, -14 respectively.

Solutions: Let us consider the cubic polynomial is ax^3+bx^2+cx+d and the values of the zeroes of the polynomials be α , β , γ .

As per the given question,

$$\alpha + \beta + \gamma = -b/a = 2/1$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = c/a = -7/1$$

$$\alpha \beta \gamma = -d/a = -14/1$$

Thus, from above three expressions we get the values of coefficient of polynomial.

$$a = 1$$
, $b = -2$, $c = -7$, $d = 14$

Hence, the cubic polynomial is $x^3-2x^2-7x+14$

3. If the zeroes of the polynomial x^3-3x^2+x+1 are a-b, a, a+b, find a and b.

Solutions: We are given with the polynomial here, p(x)

$$= x^3 - 3x^2 + x + 1$$

And zeroes are given as a - b, a, a + b

Now, comparing the given polynomial with general expression, we get;



$$px^3+qx^2+rx+s=x^3-3x^2+x+1$$

$$p = 1$$
, $q = -3$, $r = 1$ and $s = 1$

Sum of zeroes = a - b + a + a + b

$$-q/p = 3a$$

Putting the values q and p.

$$-(-3)/1 = 3a$$

a=1

Thus, the zeroes are 1-b, 1, 1+b.

Now, product of zeroes = 1(1-b)(1+b)

$$-s/p = 1-b^2$$

$$-1/1 = 1-b^2$$

$$b^2 = 1 + 1 = 2$$

$$b = \sqrt{2}$$

Hence, $1-\sqrt{2}$, 1, $1+\sqrt{2}$ are the zeroes of x_3-3x_2+x+1 .

4. If two zeroes of the polynomial x^4 - $6x^3$ - $26x^2$ +138x-35 are $2 \pm \sqrt{3}$, find other zeroes.

Solutions: Since this is a polynomial equation of degree 4, hence there will be total 4 roots.

Let
$$f(x) = x^4-6x^3-26x^2+138x-35$$

Since $2 + \sqrt{3}$ and $2 - \sqrt{3}$ are zeroes of given polynomial f(x).

$$x[x-(2+\sqrt{3})][x-(2-\sqrt{3})] = 0$$

$$(x-2-\sqrt{3})(x-2+\sqrt{3})=0$$



On multiplying the above equation we get,

 x^2-4x+1 , this is a factor of a given polynomial f(x).

Now, if we will divide f(x) by g(x), the quotient will also be a factor of f(x) and the remainder will be 0.

$$x^{2}-2x-35$$

$$x^{2}-4x+1$$

$$x^{4}-6x^{3}-26x^{2}+138x-35$$

$$x^{4}-4x^{3}+x^{2}$$

$$(\cdot) (\cdot) (\cdot)$$

$$-2x^{3}-27x^{2}+138x-35$$

$$-2x^{3}+8x^{2}-2x$$

$$(\cdot) (\cdot) (\cdot)$$

$$-35x^{2}+140x-35$$

$$-35x^{2}+140x-35$$

$$(\cdot) (\cdot) (\cdot)$$

$$0$$

So,
$$x^4-6x^3-26x^2+138x-35 = (x^2-4x+1)(x^2-2x-35)$$

Now, on further factorizing $(x^2-2x-35)$ we get,

$$x^2-(7-5)x -35 = x^2-7x+5x+35 = 0 x(x)$$

$$-7)+5(x-7)=0$$

$$(x+5)(x-7) = 0$$

So, its zeroes are given by:

$$-5$$
 and $x = 7$.

Therefore, all four zeroes of given polynomial equation are: $2+\sqrt{3}$, $2-\sqrt{3}$, -5 and 7.