## NCERT Solutions For Class 9 Maths Chapter 7 - Triangles

(Page No: 118) Exercise:

## 7.1

1. In quadrilateral $A C B D, A C=A D$ and $A B$ bisect $\angle A$ (see Fig. 7.16). Show that $\triangle A B C \cong \triangle A B D$. What can you say about BC and BD?


Fig. 7.16

## Solution:

It is given that $A C$ and $A D$ are equal i.e. $A C=A D$ and the line segment $A B$ bisects $\angle A$. We will have to now prove that the two triangles $A B C$ and $A B D$ are similari.e. $\triangle A B C \cong \triangle A B D$ Proof: Consider the triangles $\triangle A B C$ and $\triangle A B D$,
(i) $A C=A D$ (It is given in the question)
(ii) $A B=A B$ (Common)
(iii) $\angle C A B=\angle D A B$ (Since $A B$ is the bisector of angle $A$ ) So, by SAS congruency criterion, $\triangle A B C \cong$ $\triangle \mathrm{ABD}$.

For the 2 nd part of the question, $B C$ and $B D$ are of equal lengths.
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2. $A B C D$ is a quadrilateral in which $A D=B C$ and $\angle D A B=\angle C B A$ (see Fig. 7.17). Prove that
(i) $\triangle \mathrm{ABD} \cong \triangle \mathrm{BAC}$
(ii) $\mathrm{BD}=\mathrm{AC}$
(iii) $\angle A B D=\angle B A C$.

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Fig. 7.17

## Solution:

The given parameters from the questions are $\angle D A B=\angle C B A$ and $A D=B C$.
(i) $\triangle \mathrm{ABD}$ and $\triangle \mathrm{BAC}$ are similar by SAS congruency as
$\mathrm{AB}=\mathrm{BA}$ (It is the common arm)
$\angle D A B=\angle C B A$ and $A D=B C$ (These are given in the question)
So, triangles $A B D$ and $B A C$ are similari.e. $\triangle A B D \cong \triangle B A C$. (Hence proved).
(ii) It is now known that $\triangle A B D \cong \triangle B A C$ so, $B D=A C$ (by the rule of $C P C T$ ).
(iii) Since $\triangle A B D \cong \triangle B A C$ so,

Angles $\angle A B D=\angle B A C$ (by the rule of $C P C T$ ).
3. $A D$ and $B C$ are equal perpendiculars to a line segment $A B$ (see Fig. 7.18). Show that $C D$ bisects AB.


Fig. 7.18

## Solution:

It is given that $A D$ and $B C$ are two equal perpendiculars to $A B$.
We will have to prove that CD is the bisector of AB Proof:
Triangles $\triangle A O D$ and $\triangle B O C$ are similar by AAS congruency since:
(i) $\angle A=\angle B$ (They are perpendiculars)

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(ii) $\mathrm{AD}=\mathrm{BC}$ (As given in the question)
(iii) $\angle A O D=\angle B O C$ (They are vertically opposite angles) $\therefore \triangle A O D \cong \triangle B O C$.

So, $A O=O B$ ( by the rule of CPCT).
Thus, $C D$ bisects $A B$ (Hence proved).
4. I and $m$ are two parallel lines intersected by another pair of parallel linesp and q (see Fig. 7.19). Show that $\triangle A B C \cong \triangle C D A$.


Fig. 7.19

## Solution:

It is given that $\mathrm{p} \| \mathrm{q}$ and $\mathrm{I} \| \mathrm{m}$ To
prove:
Triangles $A B C$ and $C D A$ are similar i.e. $\triangle A B C \cong \triangle C D A$
Proof:
Consider the $\triangle \mathrm{ABC}$ and $\triangle \mathrm{CDA}$,
(i) $\angle B C A=\angle D A C$ and $\angle B A C=\angle D C A$ Since they are alternate interior angles
(ii) $\mathrm{AC}=\mathrm{CA}$ as it is the common arm

So, by ASA congruency criterion $\triangle A B C \cong \triangle C D A$.
5. Line $I$ is the bisector of an angle $\angle A$ and $B$ is any point on $I . B P$ and $B Q$ are perpendiculars from $B$ to the arms of $\angle A$ (see Fig. 7.20). Show that:
(i) $\triangle A P B \cong \triangle A Q B$
(ii) $\mathrm{BP}=\mathrm{BQ}$ or B is equidistant from the arms of $\angle \mathrm{A}$.


Fig. 7.20

## Solution:

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It is given that the line " $I$ " is the bisector of angle $\angle A$ and the line segments $B P$ and $B Q$ are perpendiculars drawn from l .
(i) $\triangle A P B$ and $\triangle A Q B$ are similar by AAS congruency because:
$\angle P=\angle Q$ (They are the two right angles)
$A B=A B$ (It is the common arm)
$\angle B A P=\angle B A Q$ (As line $l$ is the bisector of angle $A$ )
So, $\triangle A P B \cong \triangle A Q B$.
(ii) $B y$ the rule of $C P C T, B P=B Q$. So, it can be said the point $B$ is equidistant from the arms of $\angle A$.
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6. In Fig. 7.21, $A C=A E, A B=A D$ and $\angle B A D=\angle E A C$. Show that $B C=D E$.


Fig. 7.21

## Solution:

It is given in the question that $A B=A D, A C=A E$, and $\angle B A D=\angle E A C$
To proof:
The line segment $B C$ and $D E$ are similar i.e. $B C=D E$ Proof:
We know that $\angle B A D=\angle E A C$
Now, by adding $\angle D A C$ on both sides we get,
$\angle B A D+\angle D A C=\angle E A C+\angle D A C$
This implies, $\angle \mathrm{BAC}=\angle \mathrm{EAD}$
Now, $\triangle A B C$ and $\triangle A D E$ are similar by SAS congruency since:
(i) $A C=A E$ (As given in the question)
(ii) $\angle B A C=\angle E A D$
(iii) $A B=A D$ (It is also given in the question) $\therefore$ Triangles $A B C$ and $A D E$ are similari.e. $\triangle A B C \cong$ $\triangle \mathrm{ADE}$.

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So, by the rule of $C P C T$, it can be said that $B C=D E$.
7. $A B$ is a line segment and $P$ is its mid-point. $D$ and $E$ are points on the same side of $A B$ such that $\angle B A D=\angle A B E$ and $\angle E P A=\angle D P B$ (see Fig. 7.22). Show that
(i) $\triangle D A P \cong \triangle E B P$
(ii) $A D=B E$


Fig. 7.22

## Answer

In the question, it is given that $P$ is the mid-point of line segment $A B$. Also, $\angle B A D=\angle A B E$ and $\angle E P A=\angle D P B$
(i) It is given that $\angle E P A=\angle D P B$ Now, add $\angle D P E$ om both sides,
$\angle \mathrm{EPA}+\angle \mathrm{DPE}=\angle \mathrm{DPB}+\angle \mathrm{DPE}$
This implies that angles DPA and EPB are equal i.e. $\angle D P A=\angle E P B$
Now, consider the triangles DAP and EBP.
$\angle D P A=\angle E P B$
$A P=B P$ (Since $P$ is the mid-point of the line segement $A B$ )
$\angle B A D=\angle A B E$ (As given in the question) So, by ASA
congruency, $\triangle D A P \cong \triangle E B P$.
(ii) By the rule of $C P C T, A D=B E$.
8. In right triangle $A B C$, right angled at $C, M$ is the mid-point of hypotenuse $A B$. $C$ is joined to $M$ and produced to a point $D$ such that $D M=C M$. Point $D$ is joined to point $B$ (see Fig. 7.23). Show that:
(i) $\triangle \mathrm{AMC} \cong \triangle B M D$
(ii) $\angle D B C$ is a right angle.
(iii) $\triangle D B C \cong \triangle A C B$
(iv) $C M=1 / 2 \mathrm{AB}$

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Fig. 7.23

## Solution:

It is given that $M$ is the mid-point of the line segment $A B, \angle C=90^{\circ}$, and $D M=C M$
(i) Consider the triangles $\triangle \mathrm{AMC}$ and $\triangle \mathrm{BMD}$ :

AM = BM (Since M is the mid-point)
$C M=D M$ (Given in the question)
$\angle C M A=\angle D M B$ (They are vertically opposite angles)
So, by SAS congruency criterion, $\triangle \mathrm{AMC} \cong \triangle \mathrm{BMD}$.
(ii) $\angle A C M=\angle B D M$ (by CPCT)
$\therefore \mathrm{AC} \| \mathrm{BD}$ as alternate interior angles are equal.
Now, $\angle A C B+\angle D B C=180^{\circ}$ (Since they are co-interiors angles)
$\Rightarrow 90^{\circ}+\angle B=180^{\circ}$
$\therefore \angle \mathrm{DBC}=90^{\circ}$
(iii) $\ln \triangle D B C$ and $\triangle A C B$,
$B C=C B$ (Common side)
$\angle A C B=\angle D B C$ (They are right angles)
DB = AC (by CPCT)
So, $\triangle \mathrm{DBC} \cong \triangle \mathrm{ACB}$ by SAS congruency.
(iv) $D C=A B$ (Since $\triangle D B C \cong \triangle A C B$ )
$\Rightarrow D M=C M=A M=B M$ (Since $M$ the is mid-point)
So, $D M+C M=B M+A M$
Hence, $C M+C M=A B$
$\Rightarrow C M=(1 / 2) A B$

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## Exercise: 7.2

1. In an isosceles triangle $A B C$, with $A B=A C$, the bisectors of $\angle B$ and $\angle C$ intersect each other at O . Join A to O . Show that :
(i) $O B=O C$
(ii) AO bisects $\angle \mathrm{A}$


## Solution:

Given:
-> $A B=A C$ and
$->$ the bisectors of $\angle B$ and $\angle C$ intersect each other at $O$
(i) Since $A B C$ is an isosceles with $A B=A C$,
$=>B=\angle C$
$\Rightarrow 1 / 2 \angle B=1 / 2 \angle C$
$\Rightarrow \angle O B C=\angle O C B$ (Angle bisectors)
$\therefore \mathrm{OB}=\mathrm{OC}$ (Side opposite to the equal angles are equal.)
(ii) In $\triangle A O B$ and $\triangle A O C$,
$A B=A C$ (Given in the question)
$A O=A O$ (Common arm)
$\mathrm{OB}=\mathrm{OC}$ (As Proved Already)
So, $\triangle A O B \cong \triangle A O C$ by SSS congruence
condition. $\angle B A O=\angle C A O$ (by CPCT) Thus, AO
bisects $\angle A$.

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2. In $\triangle A B C, A D$ is the perpendicular bisector of $B C$ (see Fig. 7.30). Show that $\triangle A B C$ is an isosceles triangle in which $A B=A C$.


Fig. 7.30

## Solution:

It is given that $A D$ is the perpendicular bisector of $B C$
To prove: $A B=A C$ Proof:
In $\triangle A D B$ and $\triangle A D C$,
$A D=A D$ (It is the Common arm)
$\angle A D B=\angle A D C$
$B D=C D$ (Since $A D$ is the perpendicular bisector) So,
$\triangle A D B \cong \triangle A D C$ by SAS congruency criterion.
$A B=A C(b y C P C T)$
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3. $A B C$ is an isosceles triangle in which altitudes $B E$ and $C F$ are drawn to equal sides $A C$ and $A B$ respectively (see Fig. 7.31). Show that these altitudes are equal.


Fig. 7.31

## Solution:

Given:
(i) BE and CF are altitudes.

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(ii) $A C=A B$ To prove: $B E=C F$ Proof:

Triangles $\triangle A E B$ and $\triangle A F C$ are similar by AAS congruency since
$\angle A=\angle A$ (It is the common arm)
$\angle A E B=\angle A F C$ (They are right angles)
$A B=A C$ (Given in the question)
$\therefore \triangle A E B \cong \triangle A F C$ and so, $B E=C F$ (by CPCT).
4. $A B C$ is a triangle in which altitudes $B E$ and $C F$ to sides $A C$ and $A B$ are equal (see Fig. 7.32). Show that
(i) $\triangle \mathrm{ABE} \cong \triangle \mathrm{ACF}$
(ii) $A B=A C$, i.e., $A B C$ is an isosceles triangle.


Fig. 7.32

## Solution:

It is given that $\mathrm{BE}=\mathrm{CF}$
(i) In $\triangle A B E$ and $\triangle A C F$,
$\angle A=\angle A$ (It is the common angle)
$\angle A E B=\angle A F C$ (They are right angles)
$B E=C F$ (Given in the question)
$\therefore \triangle \mathrm{ABE} \cong \triangle \mathrm{ACF}$ by AAS congruency condition.
(ii) $A B=A C$ by $C P C T$ and so, $A B C$ is an isosceles triangle.
5. $\quad A B C$ and DBC are two isosceles triangles on the same base $B C$ (see Fig. 7.33). Show that $\angle A B D=\angle A C D$.

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Fig. 7.33

## Solution:

In the question, it is given that ABC and DBC are two isosceles triangles.
We will have to show that $\angle A B D=\angle A C D$
Proof:
Triangles $\triangle A B D$ and $\triangle A C D$ are similar by SSS congruency since
$A D=A D$ (It is the common arm)
$A B=A C$ (Since $A B C$ is an is os celes triangle) BD
$=C D$ (Since $B C D$ is an is osceles triangle) So,
$\triangle A B D \cong \triangle A C D$.
$\therefore \angle A B D=\angle A C D$ by CPCT.
6. $\triangle A B C$ is an isosceles triangle in which $A B=A C$. Side $B A$ is produced to $D$ such that $A D=$ $A B$ (see Fig. 7.34). Show that $\angle B C D$ is a right angle.


Fig. 7.34

## Solution:

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It is given that $A B=A C$ and $A D=A B$
We will have to now prove $\angle B C D$ is a right angle.

## Proof:

Consider $\triangle A B C$,
$A B=A C$ (It is given in the question)
Also, $\angle A C B=\angle A B C$ (They are angles opposite to the equal sides and so, they are equal)
Now, consider $\triangle A C D$,
$A D=A B$
Also, $\angle A D C=\angle A C D$ (They are angles opposite to the equal sides and so, they are equal)
Now,
In $\triangle A B C$,
$\angle C A B+\angle A C B+\angle A B C=180^{\circ}$
So, $\angle C A B+2 \angle A C B=180^{\circ}$
$\Rightarrow \angle C A B=180^{\circ}-2 \angle A C B---(i)$
Similarly in $\triangle \mathrm{ADC}, \angle \mathrm{CAD}=$
$180^{\circ}-2 \angle A C D--$ (ii) also,
$\angle C A B+\angle C A D=180^{\circ}$ ( $B D$ is a straight line.)
Adding (i) and (ii) we get,
$\angle C A B+\angle C A D=180^{\circ}-2 \angle A C B+180^{\circ}-2 \angle A C D$
$\Rightarrow 180^{\circ}=360^{\circ}-2 \angle A C B-2 \angle A C D$
$\Rightarrow 2(\angle A C B+\angle A C D)=180^{\circ}$
$\Rightarrow \angle B C D=90^{\circ}$
7. $A B C$ is a right-angled triangle in which $\angle A=90^{\circ}$ and $A B=A C$. Find $\angle B$ and $\angle C$.

## Solution:



In the question, it si given that
$\angle A=90^{\circ}$ and $A B=A C$
$A B=A C$

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$\Rightarrow \angle B=\angle C$ (They are angles opposite to the equal sides and so, they are equal)
Now,
$\angle A+\angle B+\angle C=180^{\circ}$ (Since the sum of the interior angles of the triangle)
$\therefore 90^{\circ}+2 \angle B=180^{\circ}$
$\Rightarrow 2 \angle B=90^{\circ}$
$\Rightarrow \angle B=45^{\circ}$
So, $\angle B=\angle C=45^{\circ}$
8. Show that the angles of an equilateral triangle are $60^{\circ}$ each.

## Solution:

Let $A B C$ be an equilateral triangle as shown below:


Here, $B C=A C=A B$ (Since the length of all sides is same)
$\Rightarrow \angle A=\angle B=\angle C$ (Sides opposite to the equal angles are equal.)
Also, we know that
$\angle A+\angle B+\angle C=180^{\circ}$
$\Rightarrow 3 \angle A=180^{\circ}$
$\Rightarrow \angle A=60^{\circ}$
$\therefore \angle A=\angle B=\angle C=60^{\circ}$
So, the angles of an equilateral triangle are always $60^{\circ}$ each.
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## Exercise: 7.3

1. $\triangle A B C$ and $\triangle D B C$ are two isosceles triangles on the same base $B C$ and vertices $A$ and $D$ are on the same side of $B C$ (see Fig. 7.39). If $A D$ is extended to intersect $B C$ at $P$, show that
(i) $\triangle A B D \cong \triangle A C D$

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(ii) $\triangle A B P \cong \triangle A C P$
(iii) $A P$ bisects $\angle A$ as well as $\angle D$.
(iv) $A P$ is the perpendicular bisector of $B C$.


Fig. 7.39

## Solution:

In the above question, it is given that $\triangle A B C$ and $\triangle D B C$ are two isosceles triangles.
(i) $\triangle \mathrm{ABD}$ and $\triangle \mathrm{ACD}$ are similar by SSS congruency because:
$A D=A D$ (It is the common arm)
$A B=A C$ (Since $\triangle A B C$ is isosceles)
$B D=C D$ (Since $\triangle D B C$ is isosceles)
$\therefore \triangle \mathrm{ABD} \cong \triangle \mathrm{ACD}$.
(ii) $\triangle A B P$ and $\triangle A C P$ are similar as:
$A P=A P$ (It is the common side)
$\angle P A B=\angle P A C($ by $C P C T$ since $\triangle A B D \cong \triangle A C D)$
$A B=A C$ (Since $\triangle A B C$ is isosceles)
So, $\triangle A B P \cong \triangle A C P$ by SAS congruency condition.
(iii) $\angle P A B=\angle P A C$ by $C P C T$ as $\triangle A B D \cong \triangle A C D$.

AP bisects $\angle A$. --- (i)
Also, $\triangle \mathrm{BPD}$ and $\triangle \mathrm{CPD}$ are similar by SSS congruency as
$\mathrm{PD}=\mathrm{PD}$ (It is the common side)
$B D=C D$ (Since $\triangle D B C$ is isosceles.)
$B P=C P($ by $C P C T$ as $\triangle A B P \cong \triangle A C P)$
So, $\triangle \mathrm{BPD} \cong \triangle \mathrm{CPD}$.
Thus, $\angle B D P=\angle C D P$ by CPCT. --- (ii)

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Now by comparing (i) and (ii) it can be said that AP bisects $\angle A$ as well as $\angle \mathrm{D}$.
(iv) $\angle \mathrm{BPD}=\angle \mathrm{CPD}$ (by CPCT as $\triangle \mathrm{BPD} \cong \triangle \mathrm{CPD}$ ) and $\mathrm{BP}=\mathrm{CP}$--- (i) also, $\angle B P D+\angle C P D=180^{\circ}$ (Since BC is a straight line.)
$\Rightarrow 2 \angle B P D=180^{\circ}$
$\Rightarrow \angle \mathrm{BPD}=90^{\circ}$---(ii)
Now, from equations (i) and (ii), it can be said that
$A P$ is the perpendicular bisector of $B C$.
2. $A D$ is an altitude of an isosceles triangle $A B C$ in which $A B=A C$. Show that
(i) $A D$ bisects $B C$
(ii) $A D$ bisects $\angle A$.

## Solution:

It is given that $A D$ is an altitude and $A B=A C$. The diagram is as follows:

(i) $\ln \triangle A B D$ and $\triangle A C D$,
$\angle A D B=\angle A D C=90^{\circ}$
$A B=A C$ (It is given in the question)
AD = AD (Common arm)
$\therefore \triangle A B D \cong \triangle A C D$ by RHS congruence
condition. Now, by the rule of $\mathrm{CPCT}, \mathrm{BD}=\mathrm{CD}$.
So, AD bisects BC
(ii)Again by the rule of $C P C T, \angle B A D=\angle C A D$ Hence, $A D$ bisects $\angle A$.
3. Two sides $A B$ and $B C$ and median $A M$ of one triangle $A B C$ are respectively equal to sides $P Q$ and QR and median PN of $\triangle P Q R$ (see Fig. 7.40). Show that:

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(i) $\triangle A B M \cong \triangle P Q N$
(ii) $\triangle A B C \cong \triangle P Q R$


Fig. 7.40

## Solution:

Given parameters are:
$A B=P Q$,
$B C=Q R$ and
$A M=P N$
(i) $1 / 2 \mathrm{BC}=\mathrm{BM}$ and $1 / 2 \mathrm{QR}=\mathrm{QN}$ (Since AM and PN are medians)

Also, $\mathrm{BC}=\mathrm{QR}$
So, $1 / 2 B C=1 / 2 Q R$
$\Rightarrow \mathrm{BM}=\mathrm{QN}$
In $\triangle A B M$ and $\triangle P Q N$,
$A M=P N$ and $A B=P Q$ (As given in the question)
$B M=Q N$ (Already proved)
$\therefore \triangle \mathrm{ABM} \cong \triangle \mathrm{PQN}$ by SSS congruency.
(ii) In $\triangle A B C$ and $\triangle P Q R$,
$A B=P Q$ and $B C=Q R$ (As given in the question)
$\angle A B C=\angle P Q R$ (by CPCT)

So, $\triangle \mathrm{ABC} \cong \triangle \mathrm{PQR}$ by SAS congruency.
4. $B E$ and CF are two equal altitudes of a triangle $A B C$. Using RHS congruence rule, prove that the triangle ABC is isosceles.


## Solution:

It is known that BE and CF are two equal altitudes.
Now, in $\triangle \mathrm{BEC}$ and $\triangle \mathrm{CFB}$,
$\angle B E C=\angle C F B=90^{\circ}$ (Same Altitudes)
$\mathrm{BC}=\mathrm{CB}$ (Common side)
$B E=C F$ (Common side)
So, $\triangle B E C \cong \triangle C F B$ by RHS congruence criterion.
Also, $\angle C=\angle B$ (by CPCT)
Therefore, $A B=A C$ as sides opposite to the equal angles is always equal.
5. $A B C$ is an isosceles triangle with $A B=A C$. Draw $A P \perp B C$ to show that $\angle B=\angle C$.

## Solution:



In the question, it is given that $A B=A C$
Now, $\triangle A B P$ and $\triangle A C P$ are similar by RHS congruency as
$\angle A P B=\angle A P C=90^{\circ}$ (AP is altitude)
$A B=A C$ (Given in the question)
$A P=A P$ (Common side) So,
$\triangle A B P \cong \triangle A C P$.
$\therefore \angle B=\angle C$ (by CPCT)
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## Exercise: 7.4

1. Show that in a right angled triangle, the hypotenuse is the longest side.


## Solution:

It is known that $A B C$ is a triangle right angled at $B$.
We know that,
$\angle A+\angle B+\angle C=180^{\circ}$
Now, if $\angle B+\angle C=90^{\circ}$ then $\angle A$ has to be $90^{\circ}$.
Since $A$ is the largest angle of the triangle, the side opposite to it must be the largest.
So, $A B$ is the hypotenuse which will be the largest side of the above right-angled triangle i.e. $\triangle A B C$.
2. In Fig. 7.48, sides $A B$ and $A C$ of $\triangle A B C$ are extended to points $P$ and $Q$ respectively. Also, $\angle P B C<\angle Q C B$. Show that $A C>A B$.


Fig. 7.48

## Solution:

It is given that $\angle \mathrm{PBC}<\angle \mathrm{QCB}$

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We know that $\angle \mathrm{ABC}+\angle \mathrm{PBC}=180^{\circ}$
So, $\angle A B C=180^{\circ}-\angle P B C$
Also,
$\angle A C B+\angle Q C B=180^{\circ}$
Therefore $\angle A C B=180^{\circ}-\angle Q C B$
Now, since $\angle P B C<\angle Q C B$,
$\therefore \angle A B C>\angle A C B$
Hence, $A C>A B$ as sides opposite to the larger angle is always larger.
3. In Fig. 7.49, $\angle B<\angle A$ and $\angle C<\angle D$. Show that $A D<B C$.


Fig. 7.49

## Solution:

In the question, it is mentioned that angles $B$ and angle $C$ is smaller than angles $A$ and $D$
respectively i.e. $\angle \mathrm{B}<\angle \mathrm{A}$ and $\angle \mathrm{C}<\angle \mathrm{D}$
Now,
Since the side opposite to the smaller angle is always smaller
AO < BO --- (i)
And OD < OC ---(ii)
By adding equation (i) and equation (ii) we get
$A O+O D<B O+O C$ So,
$A D<B C$
4. $A B$ and $C D$ are respectively the smallest and longest sides of a quadrilateral $A B C D$ (see Fig. 7.50).

Show that $\angle A>\angle C$ and $\angle B>\angle D$.


Fig. 7.50

## Solution:

In $\triangle A B D$,
$A B<A D<B D$
So, $\angle A D B<\angle A B D$--- (i) (Since angle opposite to longer side is always larger)
Now, in $\triangle B C D$,
$B C<D C<B D$
Hence, it can be concluded that
$\angle B D C<\angle C B D$--- (ii)
Now, by adding equation (i) and equation (ii) we get,
$\angle A D B+\angle B D C<\angle A B D+\angle C B D$
$=>\angle A D C<\angle A B C$
$=>\angle B>\angle D$
Similarly, In triangle $A B C$,
$\angle A C B<\angle B A C$--- (iii) (Since the angle opposite to the longer side is always larger)
Now, In $\triangle A D C$,
$\angle D C A<\angle D A C$--- (iv)
By adding equation (iii) and equation (iv) we get,
$\angle A C B+\angle D C A<\angle B A C+\angle D A C$
$\Rightarrow \angle B C D<\angle B A D$
$\therefore \angle A>\angle C$
5. In Fig 7.51, $P R>P Q$ and $P S$ bisect $\angle Q P R$. Prove that $\angle P S R>\angle P S Q$.

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Fig. 7.51

## Solution:

It is given that $P R>P Q$ and $P S$ bisects $\angle Q P R$
Now we will have to prove that angle PSR is smaller than PSQ i.e. $\angle P S R>\angle P S Q$
Proof:
$\angle Q P S=\angle R P S$--- (ii) (As PS bisects $\angle Q P R$ )
$\angle P Q R>\angle P R Q$--- (i) (Since PR $>P Q$ as angle opposite to the larger side is always larger) $\angle P S R=$ $\angle P Q R+\angle Q P S$--- (iii) (Since the exterior angle of a triangle equals to the sum of opposite interior angles)
$\angle \mathrm{PSQ}=\angle \mathrm{PRQ}+\angle \mathrm{RPS}$--- (iv) (As the exterior angle of a triangle equals to the sum of opposite interior angles)

By adding (i) and (ii)
$\angle P Q R+\angle Q P S>\angle P R Q+\angle R P S$
Now, from (i), (ii), (iii) and (iv), we get
$\angle P S R>\angle P S Q$
(Page No: 133)
6. Show that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest.

## Solution:

First, let "l" be a line segment and " $B$ " be a point lying on it. A line AB perpendicular to lis now drawn. Also, let C be any other point on I . The diagram will be as follows:


To prove: $A B$
< AC Proof:
In $\triangle \mathrm{ABC}, \angle \mathrm{B}=90^{\circ}$
Now, we know that
$\angle A+\angle B+\angle C=180^{\circ}$
$\therefore \angle A+\angle C=90^{\circ}$
Hence, $\angle C$ must be an acute angle which implies $\angle C<\angle B$
So, $A B<A C$ (As the side opposite to the larger angle is always larger)
(Page No: 133) Exercise
7.5 (Optional)

1. $A B C$ is a triangle. Locate a point in the interior of $\triangle A B C$ which is equidistant from all thevertices of $\triangle A B C$.

## Solution:

Consider a triangle $A B C$. A point in the interior of $\triangle A B C$ which is equidistant from all the vertices of $\triangle A B C$ is the "circumcenter".


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Now, the circumcenter is the point where the three perpendicular bisectors to the three sides of the triangle intersect each other. So, draw the three perpendicular bisector to $A B, B C$ and $C A$ and mark their intersecting points as O which is the required point.
2. In a triangle, locate a point in its interior which is equidistant from all the sides of thetriangle.

## Solution:

Again consider a triangle $A B C$. Now, a point in the interior of the triangle which is equidistant from all the sides of the triangle will be its "incenter".

(Here, $\angle \mathrm{BAI}=\angle \mathrm{CAI}, \angle \mathrm{ABI}=\angle \mathrm{CBI}$ and $\angle \mathrm{BCI}=\angle \mathrm{ACI}$ )
The incenter of a triangle is the intersection point where all the interior angle bisectors of the triangle meets. So, to locate incenter, draw three interior angle bisectors and mark the intersection point as O which is the required point.
3. In a huge park, people are concentrated at three points (see Fig. 7.52).

A : where there are different slides and swings for children,
$B$ : near which a man-made lake is situated, $C$ : which is
near to a large parking and exit.
Where should an icecream parlour be set up so that maximum number of persons can approach it?
(Hint : The parlour should be equidistant from $A, B$ and $C$ )
$\square$

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## Solution:

The parlor should be at a point where it is equidistant from $A B C$. If points $A B C$ are considered as a triangle, the parlor will have to be at the circumcenter as it is equidistant from the three vertices of the triangle. Thus, join $A B C$ and construct three perpendicular bisectors to $A B, B C$ and CA. Mark the intersection point as O which will be the circumcenter.
A

C
4. Complete the hexagonal and star shaped Rangolies [see Fig. (i) and (ii)] by filling them with as many equilateral triangles of side 1 cm as you can. Count the number of triangles in each case. Which has more triangles?


Fig. 7.53

## Solution:

First calculate the area of hexagon i.e. fig (i).
Hexagon Area $=6 \times \frac{25 \sqrt{3}}{4}$
Now, the area of equilateral triangles with side $1 \mathrm{~cm}=(\sqrt{ } 3 / 4) \times \mathrm{a}^{2}=(\sqrt{ } 3 / 4) \mathrm{cm}^{2}$

So, No. of equilateral triangles in fig (i) will be = (hexagon area/area of equilateral triangle) $=$ 150

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Now, in fig (ii), the star shaped rangoli will have an area as:
Area of star shape rangoli $=12 \times \frac{\sqrt{3}}{4} \times 5^{2}$
Now, the area of equilateral triangles with side $1 \mathrm{~cm}=$ (area of star shaped rangoli/area of equilateral triangle) $=300$

Thus, fig (ii) has more triangles.

