

CBSE Class 12 Maths Question Paper Solution 2015

QUESTION PAPER CODE 65/1/D

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. $p = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{8}{7}$

½ + ½ m

2. $\begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ 0 & \lambda & 3 \end{vmatrix} = 0 \Rightarrow \lambda = 7$

½ + ½ m

3. $\cos^2 \frac{\pi}{2} + \cos^2 \frac{\pi}{3} + \cos^2 \theta = 1 \Rightarrow \theta = \frac{\pi}{6}$

½ + ½ m

4. $a_{23} = \frac{|2-3|}{2} = \frac{1}{2}$

½ + ½ m

5. $\frac{dv}{dr} = -\frac{A}{r^2}, \Rightarrow r^2 \frac{d^2v}{dr^2} + 2r \frac{dv}{dr} = 0$

½ + ½ m

6. I.F. = $e^{\int \frac{1}{\sqrt{x}} dx} = e^{2\sqrt{x}}$

½ + ½ m

SECTION - B

7. Getting $A^2 = \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix}$

1½ m

$A^2 - 5A + 4I = \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix} + \begin{pmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & 5 & 0 \end{pmatrix} + \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 4 \end{pmatrix}$

1 m

$$= \begin{pmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{pmatrix} \quad 1\text{ m}$$

$$\therefore X = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & 10 \\ 5 & -4 & -2 \end{pmatrix} \quad \frac{1}{2}\text{ m}$$

OR

$$A' = \begin{pmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{pmatrix} \quad 1\text{ m}$$

$$|A'| = 1(-9) - 2(-5) = -9 + 10 = 1 \neq 0 \quad \frac{1}{2}\text{ m}$$

$$\text{Adj } A' = \begin{pmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{pmatrix} \quad 2\text{ m}$$

$$\therefore (A')^{-1} = \begin{pmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{pmatrix} \quad \frac{1}{2}\text{ m}$$

8. $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$

~~R₂~~ → R₂ - xR₁ and R₃ → R₃ - x²R₁

$$f(x) = \begin{vmatrix} a & -1 & 0 \\ 0 & a+x & -1 \\ 0 & ax+x^2 & a \end{vmatrix} \quad (\text{For bringing 2 zeroes in any row/column}) \quad 1+1\text{ m}$$

$$\therefore f(x) = a(a^2 + 2ax + x^2) = a(x+a)^2 \quad 1\text{ m}$$

$$\begin{aligned} \therefore f(2x) - f(x) &= a[2x+a]^2 - a(x+a)^2 \\ &= a x (3x+2a) \end{aligned} \quad 1\text{ m}$$

$$\begin{aligned}
 9. \quad & \int \frac{dx}{\sin x + \sin 2x} = \int \frac{dx}{\sin x (1+2 \cos x)} = \int \frac{\sin x \cdot dx}{(1-\cos x) (1+\cos x) (1+2\cos x)} & 1 \text{ m} \\
 & = - \int \frac{dt}{(1-t) (1+t) (1+2t)} \quad \text{where } \cos x = t & \frac{1}{2} \text{ m} \\
 & = \int \left(\frac{-\frac{1}{6}}{1-t} + \frac{\frac{1}{2}}{1+t} - \frac{\frac{4}{3}}{1+2t} \right) dt & 1\frac{1}{2} \text{ m} \\
 & = + \frac{1}{6} \log |1-t| + \frac{1}{2} \log |1+t| - \frac{2}{3} \log |1+2t| + c & \frac{1}{2} \text{ m} \\
 & = \frac{1}{6} \log |1-\cos x| + \frac{1}{2} \log |1+\cos x| - \frac{2}{3} \log |1+2\cos x| + c & \frac{1}{2} \text{ m}
 \end{aligned}$$

OR

$$\begin{aligned}
 & \int \frac{x^2 - 3x + 1}{\sqrt{1-x^2}} dx = \int \frac{2-3x-(1-x^2)}{\sqrt{1-x^2}} dx & \frac{1}{2} \text{ m} \\
 & = 2 \int \frac{1}{\sqrt{1-x^2}} dx - 3 \int \frac{x}{\sqrt{1-x^2}} dx - \int \sqrt{1-x^2} dx & 1 \text{ m} \\
 & = 2 \sin^{-1} x + 3\sqrt{1-x^2} - \frac{x}{2}\sqrt{1-x^2} - \frac{1}{2} \sin^{-1} x + c & (\frac{1}{2}+1+1) \text{ m} \\
 \text{or } & = \frac{3}{2} \sin^{-1} x + \frac{1}{2} (6-x)\sqrt{1-x^2} + c
 \end{aligned}$$

$$\begin{aligned}
 10. \quad I &= \int_{-\pi}^{\pi} (\cos ax - \sin bx)^2 dx = \int_{-\pi}^{\pi} (\cos^2 ax + \sin^2 bx) dx - \int_{-\pi}^{\pi} 2 \cos ax \sin bx dx \\
 &= I_1 - I_2 & \frac{1}{2} \text{ m}
 \end{aligned}$$

$$I_1 = 2 \int_0^{\pi} (\cos^2 ax + \sin^2 bx) dx \quad (\text{being an even fun.}) \quad 1 \text{ m}$$

$$I_2 = 0 \quad (\text{being an odd fun.}) \quad 1 \text{ m}$$

$$\therefore I = I_1 = \int_0^{\pi} (1 + \cos 2ax + 1 - \cos 2bx) dx \quad \frac{1}{2} \text{ m}$$

$$= \left[2x + \frac{\sin 2ax}{2a} - \frac{\sin 2bx}{2b} \right]_0^{\pi} \quad \frac{1}{2} \text{ m}$$

$$= \left[2\pi + \frac{1}{2a} \cdot \sin 2a\pi - \frac{\sin 2b\pi}{2b} \right] \text{ or } 2\pi \quad \frac{1}{2} \text{ m}$$

11. Let E_1 : selecting bag A, and E_2 : selecting bag B.

$$\therefore P(E_1) = \frac{1}{3}, P(E_2) = \frac{2}{3} \quad \frac{1}{2} + \frac{1}{2} \text{ m}$$

Let A : Getting one Red and one black ball

$$\therefore P(A|E_1) = \frac{^4C_1 \cdot ^6C_1}{^{10}C_2} = \frac{8}{15}, P(A|E_2) = \frac{^7C_1 \cdot ^3C_1}{^{10}C_2} = \frac{7}{15} \quad 1+1 \text{ m}$$

$$P(A) = P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2)$$

$$= \frac{1}{3} \cdot \frac{8}{15} + \frac{2}{3} \cdot \frac{7}{15} = \frac{22}{45} \quad 1 \text{ m}$$

OR

$$x : 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad \frac{1}{2} \text{ m}$$

$$P(x) : {}^4C_0 \left(\frac{1}{2}\right)^4 \quad {}^4C_1 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right) \quad {}^4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2 \quad {}^4C_3 \left(\frac{1}{2}\right) \left(\frac{1}{2}\right)^3 \quad {}^4C_4 \left(\frac{1}{2}\right)^4 \quad 1\frac{1}{2} \text{ m}$$

$$: = \frac{1}{16} \quad = \frac{4}{16} \quad = \frac{6}{16} \quad = \frac{4}{16} \quad = \frac{1}{16} \quad \frac{1}{2} \text{ m}$$

$$xP(x) : 0 \quad \frac{4}{16} \quad \frac{12}{16} \quad \frac{12}{16} \quad \frac{4}{16}$$

$$x^2P(x) : 0 \quad \frac{4}{16} \quad \frac{24}{16} \quad \frac{36}{16} \quad \frac{16}{16} \quad \frac{1}{2} \text{ m}$$

$$\text{Mean} = \sum x P(x) = \frac{32}{16} = 2 \quad \frac{1}{2} \text{ m}$$

$$\text{Variance} = \sum x^2 P(x) - (\sum x P(x))^2 = \frac{80}{16} - (2)^2 = 1 \quad \frac{1}{2} \text{ m}$$

$$12. \quad \vec{r} \times \hat{\vec{i}} = \left(\hat{x}\hat{i} + \hat{y}\hat{j} + \hat{z}\hat{k} \right) \hat{\vec{i}} = -\hat{y}\hat{k} + \hat{z}\hat{j} \quad 1\frac{1}{2} \text{ m}$$

$$\vec{r} \times \hat{\vec{j}} = \left(\hat{x}\hat{i} + \hat{y}\hat{j} + \hat{z}\hat{k} \right) \hat{\vec{j}} = \hat{x}\hat{k} - \hat{z}\hat{i} \quad 1\frac{1}{2} \text{ m}$$

$$\left(\vec{r} \times \hat{\vec{i}} \right), \left(\vec{r} \times \hat{\vec{j}} \right) = \left(\hat{o}\hat{i} + \hat{z}\hat{j} - \hat{y}\hat{k} \right) \cdot \left(-\hat{z}\hat{i} + \hat{o}\hat{j} + \hat{x}\hat{k} \right) = -xy \quad \frac{1}{2} \text{ m}$$

$$\left(\vec{r} \times \hat{\vec{i}} \right) \cdot \left(\vec{r} \times \hat{\vec{j}} \right) + xy = -xy + xy = 0 \quad \frac{1}{2} \text{ m}$$

13. Any point on the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ is $(3\lambda + 2, 4\lambda - 1, 12\lambda + 2)$ 1 m

If this is the point of intersection with plane $x - y + z = 5$

then $3\lambda + 2 - 4\lambda + 1 + 12\lambda + 2 - 5 = 0 \Rightarrow \lambda = 0$ 1 m

\therefore Point of intersection is $(2, -1, 2)$ 1 m

$$\text{Required distance} = \sqrt{(2+1)^2 + (-1+5)^2 + (2+10)^2} = 13 \quad 1 \text{ m}$$

14. Writing $\cot^{-1}(x+1) = \sin^{-1} \frac{1}{\sqrt{1+(x+1)^2}}$ 1½ m

$$\text{and } \tan^{-1}x = \cos^{-1} \frac{1}{\sqrt{1+x^2}} \quad 1\frac{1}{2} \text{ m}$$

$$\therefore \sin \left(\sin^{-1} \frac{1}{\sqrt{1+(x+1)^2}} \right) = \cos \left(\cos^{-1} \frac{1}{\sqrt{1+x^2}} \right) \quad \frac{1}{2} \text{ m}$$

$$1+x^2 + 2x + 1 = 1+x^2 \Rightarrow x = -\frac{1}{2} \quad \frac{1}{2} \text{ m}$$

OR

$$(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8} \Rightarrow (\tan^{-1}x)^2 + \left(\frac{\pi}{2} - \tan^{-1}x \right)^2 = \frac{5\pi^2}{8} \quad 1 \text{ m}$$

$$\therefore 2(\tan^{-1}x)^2 - \pi \tan^{-1}x - \frac{3\pi^2}{8} = 0 \quad 1\frac{1}{2} \text{ m}$$

$$\tan^{-1}x = \frac{\pi \pm \sqrt{\pi^2 + 3\pi^2}}{4} = \frac{3\pi}{4}, -\frac{\pi}{4} \quad 1 \text{ m}$$

$$\Rightarrow x = -1 \quad \frac{1}{2} \text{ m}$$

15. Putting $x^2 = \cos \theta$, we get $\frac{1}{2}$ m

$$y = \tan^{-1} \left(\frac{\sqrt{1+\cos\theta} + \sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta} - \sqrt{1-\cos\theta}} \right) $\frac{1}{2}$ m$$

$$= \tan^{-1} \left(\frac{\cos \theta/2 + \sin \theta/2}{\cos \theta/2 - \sin \theta/2} \right) = \tan^{-1} \left(\frac{1 + \tan \theta/2}{1 - \tan \theta/2} \right) 1 + $\frac{1}{2}$ m$$

$$y = \frac{\pi}{4} + \theta/2 = \frac{\pi}{4} + \frac{1}{2} \cos^{-1} x^2 $\frac{1}{2}$ m$$

$$\frac{dy}{dx} = -\frac{1}{2} \cdot \frac{1}{\sqrt{1-x^4}} \cdot 2x = -\frac{x}{\sqrt{1-x^4}} 1 m$$

16. $\frac{dx}{d\theta} = -a \sin \theta + b \cos \theta $\frac{1}{2}$ m$

$$\frac{dy}{d\theta} = a \cos \theta + b \sin \theta $\frac{1}{2}$ m$$

$$\therefore \frac{dy}{dx} = \frac{a \cos \theta + b \sin \theta}{a \sin \theta + b \cos \theta} = -\frac{x}{y} 1 $\frac{1}{2}$ m$$

$$\text{or } y \frac{dy}{dx} + x = 0 1 m$$

$$\therefore y \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \frac{dy}{dx} + 1 = 0 1 m$$

$$\text{Using (i) we get } y \frac{d^2y}{dx^2} - \frac{x}{y} \frac{dy}{dx} + 1 = 0 $\frac{1}{2}$ m$$

$$\therefore y^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = 0$$

17. Let x be the side of an equilateral triangle

$$\therefore \frac{dx}{dt} = 2 \text{ cm/s.} \quad 1 \text{ m}$$

$$\text{Area } (A) = \frac{\sqrt{3}x^2}{4} \quad 1 \text{ m}$$

$$\Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}}{2} \times \frac{dx}{dt} \quad 1 \text{ m}$$

$$\Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}}{2} \cdot (20) \cdot (2) = 20\sqrt{3} \text{ cm}^2/\text{s} \quad 1 \text{ m}$$

18. Writing $x+3 = -\frac{1}{2}(-4-2x)+1$ 1 m

$$\therefore \int (x+3)\sqrt{3-4x-x^2} dx = -\frac{1}{2} \int (-4-2x)\sqrt{3-4x-x^2} dx + \int \sqrt{7-(x+2)^2} dx \quad \frac{1}{2}+\frac{1}{2} \text{ m}$$

$$= -\frac{1}{3}(3-4x-x^2)^{\frac{3}{2}} + \frac{x+2}{2}\sqrt{3-4x-x^2} + \frac{7}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{7}}\right) + c \quad 1+1 \text{ m}$$

19.

	HF.	M	P
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$$A \begin{pmatrix} 40 & 50 & 20 \end{pmatrix} \begin{pmatrix} 25 \\ 100 \\ 50 \end{pmatrix} = \begin{pmatrix} 7000 \\ 6125 \\ 7875 \end{pmatrix} \quad 1\frac{1}{2} \text{ m}$$

$$B \begin{pmatrix} 25 & 40 & 30 \end{pmatrix} \begin{pmatrix} 25 \\ 100 \\ 50 \end{pmatrix} = \begin{pmatrix} 6125 \\ 10000 \\ 1500 \end{pmatrix}$$

$$C \begin{pmatrix} 35 & 50 & 40 \end{pmatrix} \begin{pmatrix} 25 \\ 100 \\ 50 \end{pmatrix} = \begin{pmatrix} 8750 \\ 10000 \\ 2000 \end{pmatrix}$$

Funds collected by school A : Rs. 7000,

School B : Rs. 6125, School C : Rs. 7875 1 m

Total collected : Rs. 21000 1/2 m

For writing one value 1 m

SECTION - C

20. $\forall a, b \in N, (a, b) R (a, b)$ as $ab = ba$

$\therefore R$ is reflexive (i) 2 m

Let $(a, b) R (c, d)$ for $(a, b), (c, d) \in N \times N$

$\therefore ad(b+c) = bc(a+d)$ (ii)

Also $(c, d) R (a, b) \because cb(d+a) = da(c+b)$ (using ii)

$\therefore R$ is symmetric (iii) 2 m

Let $(a, b) R (c, d)$ and $(c, d) R (e, f)$, for $a, b, c, d, e, f \in N$

$\therefore ad(b+c) = bc(a+d)$ and $cf(d+e) = de(c+f)$ 1 m

$$\therefore \frac{b+c}{bc} = \frac{a+d}{ad} \text{ and } \frac{d+e}{de} = \frac{c+f}{cf}$$

$$\text{i.e. } \frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a} \text{ and } \frac{1}{e} + \frac{1}{d} = \frac{1}{f} + \frac{1}{c}$$

$$\text{adding we get } \frac{1}{c} + \frac{1}{b} + \frac{1}{e} + \frac{1}{d} = \frac{1}{d} + \frac{1}{a} + \frac{1}{f} + \frac{1}{c}$$

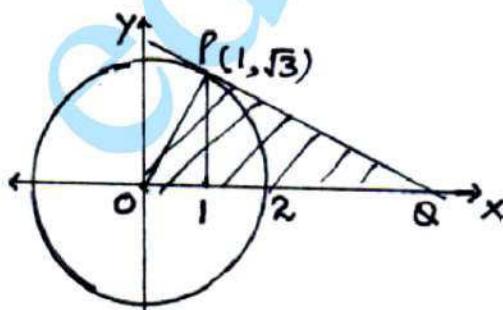
$$\Rightarrow af(b+e) = be(a+f)$$

Hence $(a, b) R (e, f) \therefore R$ is transitive (iv) ½ m

From (i), (iii) and (iv) R is an equivalence relation ½ m

21.

Correct Fig. 1 m



$$\text{Eqn. of normal (OP)} : y = \sqrt{3}x \quad \frac{1}{2} + \frac{1}{2} \text{ m}$$

Eqn. of tangent (PQ) is

$$y - \sqrt{3} = -\frac{1}{\sqrt{3}}(x - 1) \text{ i.e. } y = \frac{1}{\sqrt{3}}(4 - x) \quad 1 \text{ m}$$

Coordinates of Q (4, 0) ½ m

$$\therefore \text{Req. area} = \int_0^1 \sqrt{3x} dx + \int_1^4 \frac{1}{\sqrt{3}} (4-x) dx \quad \frac{1}{2} + \frac{1}{2} \text{ m}$$

$$= \sqrt{3} \left[\frac{x^2}{2} \right]_0^1 + \frac{1}{\sqrt{3}} \left[4x - \frac{x^2}{2} \right]_1^4 \quad 1 \text{ m}$$

$$= \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{3}} \left[16 - 8 - 4 + \frac{1}{2} \right] = 2\sqrt{3} \text{ sq. units} \quad \frac{1}{2} \text{ m}$$

OR

$$\int_1^3 (e^{2-3x} + x^2 + 1) dx \quad \text{here } h = \frac{2}{n} \quad \frac{1}{2} \text{ m}$$

$$= \lim_{h \rightarrow 0} h [f(1) + f(1+h) + f(1+2h) + \dots + f(1+(n-1)h)] \quad 1 \text{ m}$$

$$= \lim_{h \rightarrow 0} h [(e^{-1} + 2) + (e^{-1-3h} + 2 + 2h + h^2) + (e^{-1-6h} + 2 + 4h + 4h^2) + \dots + (e^{-1-3(n-1)h} + 2 + 2(n-1)h + (n-1)^2 h^2)] \quad 1 \text{ m}$$

$$= \lim_{h \rightarrow 0} h [e^{-1}(1 + e^{-3h} + e^{-6h} + \dots + e^{-3(n-1)h}) + 2n + 2h(1+2+\dots+(n-1)) + h^2(1^2 + 2^2 + \dots + (n-1)^2)] \quad 1 \frac{1}{2} \text{ m}$$

$$= \lim_{h \rightarrow 0} h \left(e^{-1} \cdot \frac{e^{-3nh} - 1}{e^{-3n} - 1} \cdot h + 2nh + 2 \frac{nh(nh-h)}{2} + \frac{nh(nh-h)(2nh-h)}{6} \right) \quad 1 \text{ m}$$

$$= e^{-1} \cdot \frac{(e^{-6} - 1)}{-3} + 4 + 4 + \frac{8}{3} = -e^{-1} \frac{(e^{-6} - 1)}{3} + \frac{32}{3} \quad 1 \text{ m}$$

22. Given differential equation can be written as

$$\frac{dx}{dy} + \frac{1}{1+y^2} \cdot x = \frac{\tan^{-1}y}{1+y^2} \quad 1\text{ m}$$

∴ Integrating factor is $e^{\tan^{-1}y}$ 1 m

$$\therefore \text{Solution is : } x \cdot e^{\tan^{-1}y} = \int \frac{\tan^{-1}y \cdot e^{\tan^{-1}y}}{1+y^2} dy \quad 1\frac{1}{2} \text{ m}$$

$$\Rightarrow x \cdot e^{\tan^{-1}y} = \int t e^t dt \text{ where } \tan^{-1}y = t \quad 1 \text{ m}$$

$$= t e^t - e^t + c = e^{\tan^{-1}y} (\tan^{-1}y - 1) + c \quad 1\frac{1}{2} \text{ m}$$

$$\text{or } x = \tan^{-1}y - 1 + c e^{-\tan^{-1}y}$$

OR

$$\text{Given differential equation is } \frac{dy}{dx} = \frac{y/x}{1+(y/x)^2}$$

$$\text{Putting } \frac{y}{x} = v \text{ to get } v + x \frac{dv}{dx} = \frac{v}{1+v^2} \quad 1\frac{1}{2} \text{ m}$$

$$\therefore x \frac{dv}{dx} = \frac{v}{1+v^2} - v = \frac{-v^3}{1+v^2} \quad 1\frac{1}{2} \text{ m}$$

$$\Rightarrow \int \frac{v^2+1}{v^3} dv = - \int \frac{dx}{x} \quad \frac{1}{2} \text{ m}$$

$$\Rightarrow \log|v| - \frac{1}{2v^2} = - \log|x| + c \quad 1 \text{ m}$$

$$\therefore \log y - \frac{x^2}{2y^2} = c \quad 1 \text{ m}$$

$$x=0, y=1 \Rightarrow c=0 \therefore \log y - \frac{x^2}{2y^2} = 0 \quad \frac{1}{2} \text{ m}$$

23. Any point on line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ is $(2\lambda+1, 3\lambda-1, 4\lambda+1)$ 1 m

$$\therefore \frac{2\lambda+1-3}{1} = \frac{3\lambda-1-k}{2} = \frac{4\lambda+1}{1} \Rightarrow \lambda = -\frac{3}{2}, \text{ hence } k = \frac{9}{2} \quad 2\frac{1}{2} \text{ m}$$

Eqn. of plane containing three lines is

$$\begin{vmatrix} x-1 & y+1 & z-1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0 \quad 1 \text{ m}$$

$$\Rightarrow -5(x-1) + 2(y+1) + 1(z-1) = 0 \quad 1 \text{ m}$$

$$\text{i.e. } 5x - 2y - z - 6 = 0 \quad \frac{1}{2} \text{ m}$$

24. $P(\overline{A} \cap B) = \frac{2}{15} \Rightarrow P(\overline{A}) \cdot P(B) = \frac{2}{15} \quad 1 \text{ m}$

$$P(A \cap \overline{B}) = \frac{1}{6} \Rightarrow P(A) \cdot P(\overline{B}) = \frac{1}{6} \quad 1 \text{ m}$$

$$\therefore (1 - P(A))P(B) = \frac{2}{15} \text{ or } P(B) - P(A) \cdot P(B) = \frac{2}{15} \dots \text{(i)} \quad 1 \text{ m}$$

$$P(A)(1 - P(B)) = \frac{1}{6} \text{ or } P(A) - P(A) \cdot P(B) = \frac{1}{6} \dots \text{(ii)} \quad 1 \text{ m}$$

$$\text{From (i) and (ii)} \quad P(A) - P(B) = \frac{1}{6} - \frac{2}{15} = \frac{1}{30} \quad \frac{1}{2} \text{ m}$$

$$\text{Let } P(A) = x, P(B) = y \quad \therefore x = \left(\frac{1}{30} + y \right)$$

$$\text{(i)} \Rightarrow y - \left(\frac{1}{30} + y \right) y = \frac{2}{15} \quad \therefore 30y^2 - 29y + 4 = 0 \quad \frac{1}{2} \text{ m}$$

$$\text{Solving to get } y = \frac{1}{6} \text{ or } y = \frac{4}{5}$$

$$\therefore x = \frac{1}{5} \text{ or } x = \frac{5}{6} \quad \frac{1}{2} \text{ m}$$

$$\text{Hence } P(A) = \frac{1}{5}, P(B) = \frac{1}{6} \quad \text{OR} \quad P(A) = \frac{5}{6}, P(B) = \frac{4}{5} \quad \frac{1}{2} \text{ m}$$

25. $f(x) = \sin x - \cos x, 0 < x < 2\pi$

$f'(x) = 0 \Rightarrow \cos x + \sin x = 0 \text{ or } \tan x = -1,$

1 m

$\therefore x = \frac{3\pi}{4}, \frac{7\pi}{4}$

1 m

$f''(x) = \cos x - \sin x$

1 m

$f''\left(\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}$ i.e - ve so, $x = \frac{3\pi}{4}$ is Local Maxima

1 m

and $f''\left(\frac{7\pi}{4}\right) = -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$ i.e + ve so, $x = \frac{7\pi}{4}$ is Local Minima

1 m

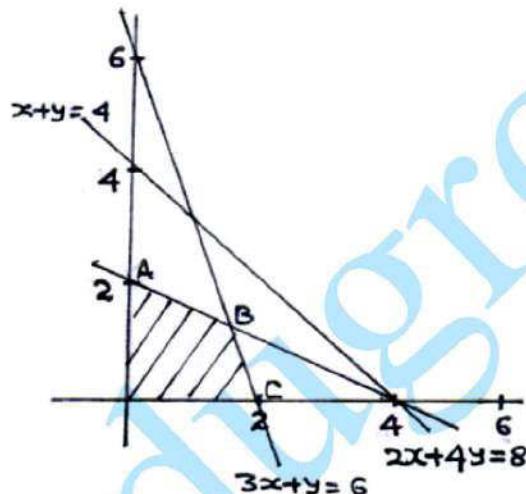
Local Maximum value $= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$

½ m

Local Minimum value $= -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = -\sqrt{2}$

½ m

26.



Correct graphs of three lines

1x3 = 3 m

Correctly shading

1 m

feasible region

Vertices are

A (0, 2), B (1.6, 1.2), C (2, 0)

1 m

$Z = 2x + 5y$ is maximum

at A (0, 2) and maximum value = 10

1 m