

CBSE Class 12 Maths Question Paper Solution 2019

QUESTION PAPER CODE 65/1/1 EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $AB = 2I \Rightarrow |AB| = |2I| \Rightarrow |A| \cdot |B| = 2^3|I|$ $\frac{1}{2}$

$$\Rightarrow 2 \times |B| = 8 \Rightarrow |B| = 4$$
 $\frac{1}{2}$

2. $(f \circ f)(x) = f(x + 1) = x + 2$ $\frac{1}{2}$

$$\frac{d}{dx}(f \circ f)(x) = 1$$
 $\frac{1}{2}$

3. order = 2, degree = 1 $\frac{1}{2} + \frac{1}{2}$

4. d.c.'s = $\langle \cos 90^\circ, \cos 135^\circ, \cos 45^\circ \rangle$ $\frac{1}{2}$

$$= \left\langle 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$$
 $\frac{1}{2}$

OR

$$\vec{r} = (3\hat{i} + 4\hat{j} + 5\hat{k}) + \lambda(2\hat{i} + 2\hat{j} - 3\hat{k})$$
 1

SECTION B

5. As $a, b \in R \Rightarrow ab \in R \Rightarrow ab + 1 \in R \Rightarrow a * b \in R \Rightarrow *$ is binary. 1

$$\text{For associative } (a * b) * c = (ab + 1) * c = (ab + 1)c + 1 = abc + c + 1$$

$$\text{also, } a * (b * c) = a * (bc + 1) = a(bc + 1) + 1 = abc + a + 1$$

$$\text{In general } (a * b) * c \neq a * (b * c) \Rightarrow *$$
 is not associative. 1

6. $2A - \begin{bmatrix} -6 & 6 & 0 \\ 9 & 3 & 12 \end{bmatrix} + \begin{bmatrix} 10 & 0 & -10 \\ 35 & 5 & 30 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ 1

$$\Rightarrow A = \begin{bmatrix} -8 & 3 & 5 \\ -13 & -1 & -9 \end{bmatrix}$$
 1

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7. Put $\tan x = t \Rightarrow \sec^2 x dx = dt$ $\frac{1}{2}$

$$I = \int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}} dx = \int \frac{dt}{\sqrt{t^2 + 4}} = \log |t + \sqrt{t^2 + 4}| + C$$
1

$$= \log |\tan x + \sqrt{\tan^2 x + 4}| + C$$
 $\frac{1}{2}$

8. Let $I = \int \sqrt{1 - \sin 2x} dx$

$$= \int (\sin x - \cos x) dx \quad \text{as } \sin x > \cos x \text{ when } x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$$
1

$$= -\cos x - \sin x + C$$
1

OR

$$I = \int \sin^{-1}(2x) dx$$
1

$$= x \cdot \sin^{-1}(2x) - \int \frac{2x}{\sqrt{1-4x^2}} dx$$
1

$$= x \cdot \sin^{-1}(2x) + \frac{1}{4} \int \frac{-8x}{\sqrt{1-4x^2}} dx = x \sin^{-1}(2x) + \frac{1}{2} \sqrt{1-4x^2} + C$$
1

9. $y' = be^{2x} + 2y \Rightarrow b = \frac{y' - 2y}{e^{2x}}$ $\frac{1}{2}$

differentiating again

$$\frac{e^{2x} \cdot (y'' - 2y') - (y' - 2y) \cdot 2e^{2x}}{(e^{2x})^2} = 0$$
1

$$\Rightarrow y'' - 4y' + 4y = 0 \text{ or } \frac{d^2y}{dx^2} - 4 \frac{dy}{dx} + 4y = 0$$
 $\frac{1}{2}$

10. Given $|\hat{a} + \hat{b}| = 1$

$$\text{As } |\hat{a} + \hat{b}|^2 + |\hat{a} - \hat{b}|^2 = 2(|\hat{a}|^2 + |\hat{b}|^2)$$
1

$$\Rightarrow 1 + |\vec{a} - \vec{b}|^2 = 2(1+1)$$

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$$\Rightarrow |\hat{a} - \hat{b}|^2 = 3 \Rightarrow |\hat{a} - \hat{b}| = \sqrt{3}$$

1

OR

$$[\vec{a} \quad \vec{b} \quad \vec{c}] = \begin{vmatrix} 2 & 3 & 1 \\ 1 & -2 & 1 \\ -3 & 1 & 2 \end{vmatrix}$$

1

$$= -30$$

1

11. $A = \{2, 4, 6\}, B = \{1, 2, 3\}, A \cap B = \{2\}$

$$\text{Now, } P(A) = \frac{1}{2}, P(B) = \frac{1}{2}, P(A \cap B) = \frac{1}{6}$$

1

$$\text{as } P(A) \times P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \neq P(A \cap B)$$

$\frac{1}{2}$

$\Rightarrow A$ and B are not independent.

$\frac{1}{2}$

12. Let X : getting an odd number

$$p = \frac{1}{2}, q = \frac{1}{2}, n = 6$$

$\frac{1}{2}$

$$(i) P(X = 5) = {}^6C_5 \left(\frac{1}{2}\right)^6 = \frac{3}{32}$$

$\frac{1}{2}$

$$(ii) P(X \leq 5) = 1 - P(X = 6) = 1 - \frac{1}{64} = \frac{63}{64}$$

1

OR

$$k + 2k + 3k = 1$$

1

$$\Rightarrow k = \frac{1}{6}$$

1

13. Clearly $a \leq a \forall a \in \mathbb{R} \Rightarrow (a, a) \in R \Rightarrow R$ is reflexive.

1

For transitive:

Let $(a, b) \in R$ and $(b, c) \in R, a, b, c \in \mathbb{R}$

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$$\Rightarrow a \leq b \text{ and } b \leq c \Rightarrow a \leq c \Rightarrow (a, c) \in R$$

 $\Rightarrow R$ is transitive.

 $1\frac{1}{2}$

For non-symmetric:

Let $a = 1, b = 2$. As $1 \leq 2 \Rightarrow (1, 2) \in R$ but $2 \not\leq 1 \Rightarrow (2, 1) \notin R$

 $\Rightarrow R$ is non-symmetric.

 $1\frac{1}{2}$

OR

 For one-one. Let $x_1, x_2 \in N$.

$$f(x_1) = f(x_2) \Rightarrow x_1^2 + x_1 + 1 = x_2^2 + x_2 + 1$$

$$\Rightarrow (x_1 - x_2)(x_1 + x_2 + 1) = 0$$

$$\Rightarrow x_1 = x_2 \text{ as } x_1 + x_2 + 1 \neq 0$$

 $(\because x_1, x_2 \in N)$
 $1\frac{1}{2}$
 $\Rightarrow f$ is one-one.

For not onto.

for $y = 1 \in N$, there is no $x \in N$ for which $f(x) = 1$

 $1\frac{1}{2}$

$$\text{For } f^{-1}: y = f(x) \Rightarrow y = x^2 + x + 1 \Rightarrow y = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$$

$$\Rightarrow x = \frac{\sqrt{4y-3}-1}{2}$$

$$\therefore f^{-1}(y) = \frac{\sqrt{4y-3}-1}{2} \text{ or } f^{-1}(x) = \frac{\sqrt{4x-3}-1}{2}$$

 1

$$14. \quad \tan^{-1} \left(\frac{4x+6x}{1-(4x)(6x)} \right) = \frac{\pi}{4}$$

 1

$$\Rightarrow \frac{10x}{1-24x^2} = 1 \Rightarrow 24x^2 + 10x - 1 = 0$$

 $1\frac{1}{2}$

$$\Rightarrow x = \frac{1}{12} \text{ or } -\frac{1}{2}$$

 1

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as $x = -\frac{1}{2}$ does not satisfy the given equation, so $x = \frac{1}{12}$ 1

15. LHS = $\begin{vmatrix} a^2 + 2a & 2a + 1 & 1 \\ 2a + 1 & a + 2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$

$$R_1 \rightarrow R_1 - R_2, R_2 \rightarrow R_2 - R_3$$

$$= \begin{vmatrix} a^2 - 1 & a - 1 & 0 \\ 2(a-1) & a - 1 & 0 \\ 3 & 3 & 1 \end{vmatrix} \quad 2$$

$$= (a-1)^2 \begin{vmatrix} a+1 & 1 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{vmatrix} \quad 1$$

Expanding along C_3 ,

$$= (a-1)^2 \cdot (a-1) = (a-1)^3 = RHS. \quad 1$$

16. $\log(x^2 + y^2) = 2 \tan^{-1}\left(\frac{y}{x}\right)$

differentiating both sides w.r.t. x,

$$\frac{1}{x^2 + y^2} \left(2x + 2y \frac{dy}{dx} \right) = 2 \cdot \frac{1}{1 + \left(\frac{y}{x}\right)^2} \cdot \left(\frac{x \cdot \frac{dy}{dx} - y \cdot 1}{x^2} \right) \quad 2$$

$$\Rightarrow \frac{2}{x^2 + y^2} \left(x + y \frac{dy}{dx} \right) = \frac{2x^2}{x^2 + y^2} \cdot \frac{1}{x^2} \cdot \left(x \frac{dy}{dx} - y \right) \quad 1$$

$$\Rightarrow (x+y) = (x-y) \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{x+y}{x-y} \quad 1$$

OR

Let $u = x^y, v = y^x$. Then $u - v = a^b$

$$\Rightarrow \frac{du}{dx} - \frac{dv}{dx} = 0 \quad \dots(1) \quad 1$$

Now, $\log u = y \cdot \log x$

$$\Rightarrow \frac{1}{u} \cdot \frac{du}{dx} = y \cdot \frac{1}{x} + \log x \cdot \frac{dy}{dx} \Rightarrow \frac{du}{dx} = x^y \left(\frac{y}{x} + \log x \cdot \frac{dy}{dx} \right) \quad \dots(2) \quad 1$$

Again, $\log v = x \cdot \log y$

$$\Rightarrow \frac{1}{v} \cdot \frac{dv}{dx} = x \cdot \frac{1}{y} \cdot \frac{dy}{dx} + \log y \cdot 1 \Rightarrow \frac{dv}{dx} = y^x \left(\frac{x}{y} \frac{dy}{dx} + \log y \right) \quad \dots(3) \quad 1$$

From (1), (2) and (3)

$$x^y \left(\frac{y}{x} + \log x \frac{dy}{dx} \right) - y^x \left(\frac{x}{y} \frac{dy}{dx} + \log y \right) = 0 \quad \frac{1}{2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y^x \cdot \log y - x^{y-1} \cdot y}{x^y \cdot \log x - y^{x-1} \cdot x} \quad \frac{1}{2}$$

17. $y = (\sin^{-1} x)^2$

$$\Rightarrow y' = 2 \cdot \sin^{-1} x \cdot \frac{1}{\sqrt{1-x^2}} \quad 1$$

$$\Rightarrow \sqrt{1-x^2} \cdot y' = 2 \sin^{-1} x$$

$$\Rightarrow \sqrt{1-x^2} \cdot y'' + y' \cdot \frac{1}{2\sqrt{1-x^2}} (-2x) = \frac{2}{\sqrt{1-x^2}} \quad 2$$

$$\Rightarrow (1-x^2) \cdot y'' - xy' = 2 \text{ or } (1-x^2) \cdot \frac{d^2y}{dx^2} - x \frac{dy}{dx} - 2 = 0. \quad 1$$

18. Let the point of contact be $P(x_1, y_1)$

$$\frac{dy}{dx} = \frac{3}{2\sqrt{3x-2}} \quad (\text{slope of tangent})$$

$$\Rightarrow m_1 = \left. \frac{dy}{dx} \right|_{(x_1, y_1)} = \frac{3}{2\sqrt{3x_1-2}} \quad 1$$

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 also, slope of given line = 2 = m_2

$$m_1 = m_2 \Rightarrow x_1 = \frac{41}{48}$$

$$\text{when } x_1 = \frac{41}{48}, y_1 = \sqrt{\frac{41}{16} - 2} = \frac{3}{4} \quad \therefore P\left(\frac{41}{48}, \frac{3}{4}\right)$$

$$\text{Equation of tangent is: } y - \frac{3}{4} = 2\left(x - \frac{41}{48}\right)$$

$$\Rightarrow 48x - 24y = 23$$

$$\text{and, Equation of normal is: } y - \frac{3}{4} = -\frac{1}{2}\left(x - \frac{41}{48}\right)$$

$$\Rightarrow 48x + 96y = 113$$

$$19. I = \int \frac{3x+5}{x^2+3x-18} dx = \frac{3}{2} \int \frac{2x+3}{x^2+3x-18} dx + \frac{1}{2} \int \frac{1}{x^2+3x-18} dx$$

$$= \frac{3}{2} \int \frac{2x+3}{x^2+3x-18} dx + \frac{1}{2} \int \frac{1}{\left(x + \frac{3}{2}\right)^2 - \left(\frac{9}{2}\right)^2} dx$$

$$= \frac{3}{2} \log|x^2 + 3x - 18| + \frac{1}{18} \log \left| \frac{x-3}{x+6} \right| + C$$

$$20. \text{ Let } I = \int_0^a f(a-x) dx$$

$$\text{Put } a-x = t \Rightarrow -dx = dt$$

$$I = - \int_a^0 f(t) dt = \int_0^a f(t) dt = \int_0^a f(x) dx$$

II part.

$$I = \int_0^\pi \frac{x \sin x}{1+\cos^2 x} dx$$

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$$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x) \cdot \sin x}{1 + \cos^2 x} dx$$

$$\Rightarrow 2I = \int_0^{\pi} \frac{\pi \cdot \sin x}{1 + \cos^2 x} dx$$

Put $\cos x = t \Rightarrow -\sin x dx = dt$

$$\Rightarrow I = -\frac{\pi}{2} \cdot \int_{-1}^{1} \frac{dt}{1+t^2} = \frac{\pi}{2} \times 2 \times \int_0^1 \frac{dt}{1+t^2}$$

$$= \pi [\tan^{-1} t]_0^1 = \frac{\pi^2}{4}$$

$$21. \text{ Writing } \frac{dy}{dx} = \frac{y + \sqrt{x^2 + y^2}}{x} = \frac{y}{x} + \sqrt{1 + \left(\frac{y}{x}\right)^2}$$

$$\text{Put } y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$\text{Differential equation becomes } v + x \frac{dv}{dx} = v + \sqrt{1+v^2}$$

$$\Rightarrow \int \frac{dv}{\sqrt{1+v^2}} = \int \frac{dx}{x}$$

$$\Rightarrow \log |v + \sqrt{1+v^2}| = \log |x| + \log c$$

$$\Rightarrow v + \sqrt{1+v^2} = cx \Rightarrow y + \sqrt{x^2 + y^2} = cx^2$$

$$\text{when } x = 1, y = 0 \Rightarrow c = 1$$

$$\therefore y + \sqrt{x^2 + y^2} = x^2$$

OR

$$\text{Given equation is } \frac{dy}{dx} + \frac{2x}{1+x^2} \cdot y = \frac{4x^2}{1+x^2}$$

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$$\text{I.F.} = e^{\int \frac{2x}{1+x^2} dx} = 1+x^2$$
1

Solution is given by,

$$y \cdot (1+x^2) = \int \frac{4x^2}{1+x^2} \cdot (1+x^2) dx = \int 4x^2 dx$$
1

$$\Rightarrow y \cdot (1+x^2) = \frac{4x^3}{3} + c$$
1/2

$$\text{when } x = 0, y = 0 \Rightarrow c = 0$$
1/2

$$y \cdot (1+x^2) = \frac{4x^3}{3} \text{ or } y = \frac{4x^3}{3(1+x^2)}$$
1/2

22. $\overrightarrow{AB} = \hat{i} + 4\hat{j} - \hat{k}$

1

$$\overrightarrow{CD} = -2\hat{i} - 8\hat{j} + 2\hat{k}$$
1

Let required angle be θ .

$$\text{Then } \cos \theta = \frac{\overrightarrow{AB} \cdot \overrightarrow{CD}}{|\overrightarrow{AB}| |\overrightarrow{CD}|} = \frac{-2 - 32 - 2}{\sqrt{18} \sqrt{72}} = -1$$
1

$$\Rightarrow \theta = 180^\circ \text{ or } \pi$$
1/2

Since $\theta = \pi$ so \overrightarrow{AB} and \overrightarrow{CD} are collinear.

1/2

23. Given lines are: $\frac{x-1}{-3} = \frac{y-2}{\left(\frac{\lambda}{7}\right)} = \frac{z-3}{2}$ and $\frac{x-1}{\left(\frac{-3\lambda}{7}\right)} = \frac{y-5}{1} = \frac{z-6}{-5}$

1

As lines are perpendicular,

$$(-3)\left(\frac{-3\lambda}{7}\right) + \left(\frac{\lambda}{7}\right)(1) + 2(-5) = 0 \Rightarrow \lambda = 7$$
1

So, lines are

$$\frac{x-1}{-3} = \frac{y-2}{1} = \frac{z-3}{2} \text{ and } \frac{x-1}{-3} = \frac{y-5}{1} = \frac{z-6}{-5}$$
1/2

65/1/1

$$\text{Consider } \Delta = \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 0 & 3 & 3 \\ -3 & 1 & 2 \\ -3 & 1 & -5 \end{vmatrix} = -63 \quad 1$$

as $\Delta \neq 0 \Rightarrow$ lines are not intersecting. $\frac{1}{2}$

SECTION D

24. $|A| = 4 \neq 0 \Rightarrow A^{-1}$ exists. $\frac{1}{2}$

$$\text{adj } A = \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix} \quad 2$$

$$\therefore A^{-1} = \frac{1}{|A|} \cdot \text{adj } A = \frac{1}{4} \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix} \quad \frac{1}{2}$$

Given system of equations can be written as $AX = B$ where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$

$$\therefore X = A^{-1} \cdot B \quad 1$$

$$= \frac{1}{4} \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} \quad 1$$

$$\Rightarrow x = 3, y = 1, z = 2 \quad \frac{1}{2}$$

OR

$$A = I \cdot A$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A \quad 1$$

$$\left. \begin{array}{l}
 R_2 \rightarrow R_2 + R_1 \\
 \Rightarrow \begin{bmatrix} 1 & 2 & -2 \\ 0 & 5 & -2 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A \\
 R_2 \rightarrow \frac{R_2}{5} \\
 \Rightarrow \begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & -2/5 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1/5 & 1/5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A \\
 R_1 \rightarrow R_1 - 2R_2, R_3 \rightarrow R_3 + 2R_2 \\
 \Rightarrow \begin{bmatrix} 1 & 0 & -6/5 \\ 0 & 1 & -2/5 \\ 0 & 0 & 1/5 \end{bmatrix} = \begin{bmatrix} 3/5 & -2/5 & 0 \\ 1/5 & 1/5 & 0 \\ 2/5 & 2/5 & 1 \end{bmatrix} \cdot A \\
 R_3 \rightarrow 5R_3 \\
 \Rightarrow \begin{bmatrix} 1 & 0 & -6/5 \\ 0 & 1 & -2/5 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3/5 & -2/5 & 0 \\ 1/5 & 1/5 & 0 \\ 2 & 2 & 5 \end{bmatrix} \cdot A \\
 R_1 \rightarrow R_1 + \frac{6}{5}R_3, R_2 \rightarrow R_2 + \frac{2}{5}R_3 \\
 \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} \cdot A \\
 \Rightarrow A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}
 \end{array} \right\}$$

4

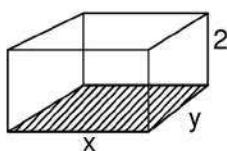
1

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25.

$$V = 2xy \Rightarrow 2xy = 8 \text{ (given)}$$

1



$$\Rightarrow y = \frac{4}{x}$$

$$\text{Now, cost, } C = 70xy + 45 \times 2 \times (2x + 2y)$$

1

$$= 280 + 180x + \frac{720}{x}$$

1

$$\frac{dC}{dx} = 180 - \frac{720}{x^2}$$

1

$$\frac{dC}{dx} = 0 \Rightarrow x = 2m$$

$\frac{1}{2}$

$$\frac{d^2C}{dx^2} = \frac{1440}{x^3} = 180 > 0 \text{ at } x = 2$$

$\frac{1}{2}$

$\Rightarrow C$ is minimum at $x = 2m$.

$\frac{1}{2}$

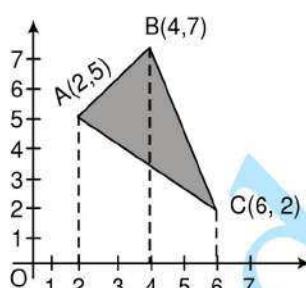
$$\text{Minimum cost} = 280 + 180(2) + \frac{720}{2} = ₹ 1,000$$

$\frac{1}{2}$

26.

Correct Figure

1



$$\left. \begin{array}{l} \text{Equation of AB: } y = x + 3 \\ \text{Equation of BC: } y = \frac{-5x}{2} + 17 \\ \text{Equation of AC: } y = \frac{-3x}{4} + \frac{13}{2} \end{array} \right\}$$

$1\frac{1}{2}$

$$\text{Required Area} = \int_2^4 (x+3) dx + \int_4^6 \left(\frac{-5x}{2} + 17 \right) dx - \int_2^6 \left(\frac{-3x}{4} + \frac{13}{2} \right) dx$$

$1\frac{1}{2}$

$$= \left[\frac{(x+3)^2}{2} \right]_2^4 + \left[\frac{-5x^2}{4} + 17x \right]_4^6 - \left[\frac{-3x^2}{8} + \frac{13x}{2} \right]_2^6$$

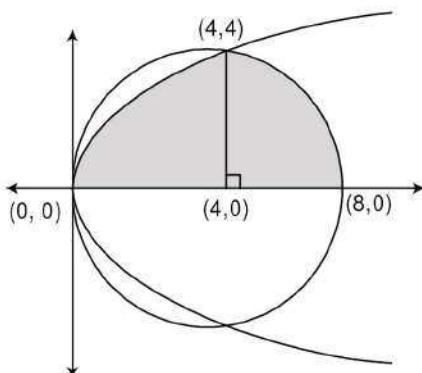
$1\frac{1}{2}$

$$= 7$$

$\frac{1}{2}$

65/1/1

OR



Correct Figure

1

$$\text{Given circle } x^2 - 8x + y^2 = 0$$

$$\text{or } (x-4)^2 + y^2 = 4^2$$

Point of intersection (0, 0) and (4, 4)

1

$$\text{Required Area} = \int_0^4 2\sqrt{x} \, dx + \int_4^8 \sqrt{4^2 - (x-4)^2} \, dx$$

1 1/2

$$= \left[\frac{4}{3}x^{3/2} \right]_0^4 + \left[\frac{x-4}{2}\sqrt{16-(x-4)^2} + \frac{16}{2} \sin^{-1}\left(\frac{x-4}{4}\right) \right]_4^8$$

$$= \left(4\pi + \frac{32}{3} \right)$$

1 1/2

27. Equation of plane is $\begin{vmatrix} x-2 & y-2 & z+1 \\ 1 & 2 & 3 \\ 5 & -2 & 7 \end{vmatrix} = 0$

2

$$\Rightarrow 5x + 2y - 3z = 17 \quad (\text{Cartesian equation})$$

1

$$\text{Vector equation is } \vec{r} \cdot (5\hat{i} + 2\hat{j} - 3\hat{k}) = 17$$

1

Equation of required parallel plane is

$$\vec{r} \cdot (5\hat{i} + 2\hat{j} - 3\hat{k}) = (4\hat{i} + 3\hat{j} + \hat{k}) \cdot (5\hat{i} + 2\hat{j} - 3\hat{k})$$

1

$$\Rightarrow \vec{r} \cdot (5\hat{i} + 2\hat{j} - 3\hat{k}) = 23$$

1

OR

$$\text{Let required plane be } a(x+1) + b(y-3) + c(z+4) = 0 \quad \dots(1)$$

1

Plane contains the given line, so it will also contain the point (1, 1, 0).

$$\text{So, } 2a - 2b + 4c = 0 \quad \text{or} \quad a - b + 2c = 0$$

... (2)

1

$$\text{Also, } a + 2b - c = 0$$

... (3)

1

65/1/1

From (2) and (3),

$$\frac{a}{-3} = \frac{b}{3} = \frac{c}{3}$$

1

\therefore Required plane is $-3(x + 1) + 3(y - 3) + 3(z + 4) = 0$

$$\therefore -x + y + z = 0$$

Also vector equation is: $\vec{r} \cdot (-\hat{i} + \hat{j} + \hat{k}) = 0$

1

$$\text{Length of perpendicular from } (2, 1, 4) = \frac{|-2+1+4|}{\sqrt{(-1)^2 + 1^2 + 1^2}} = \sqrt{3}$$

1

28.

$$\left. \begin{array}{l} E_1 : \text{item is produced by A} \\ E_2 : \text{item is produced by B} \\ E_3 : \text{item is produced by C} \\ A : \text{defective item is found.} \end{array} \right\}$$

1

$$P(E_1) = \frac{50}{100}, P(E_2) = \frac{30}{100}, P(E_3) = \frac{20}{100}$$

1

$$P(A | E_1) = \frac{1}{100}, P(A | E_2) = \frac{5}{100}, P(A | E_3) = \frac{7}{100}$$

1

$$P(E_1 | A) = \frac{\frac{50}{100} \times \frac{1}{100}}{\frac{50}{100} \times \frac{1}{100} + \frac{30}{100} \times \frac{5}{100} + \frac{20}{100} \times \frac{7}{100}}$$

2

$$= \frac{5}{34}$$

1

65/1/1
29.

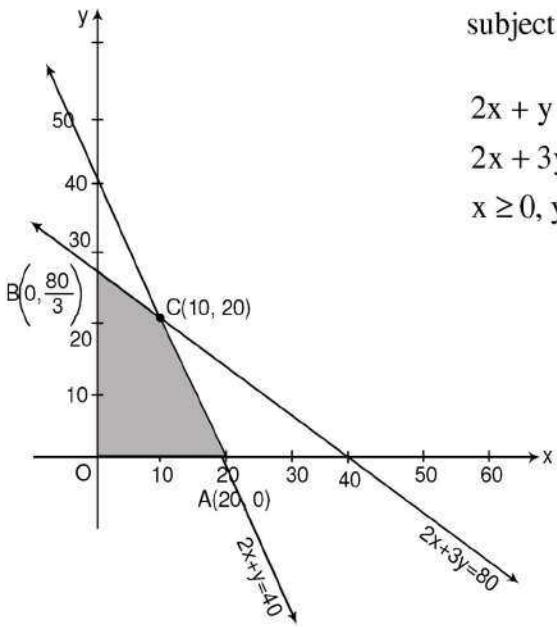
Let number of items produced of model A be x and that of model B be y .

LPP is:

$$\text{Maximize, profit } z = 15x + 10y \quad 1$$

subject to

$$\left. \begin{array}{l} 2x + y \leq 5(8) \quad \text{i.e., } 2x + y \leq 40 \\ 2x + 3y \leq 10(8) \quad \text{i.e., } 2x + 3y \leq 80 \\ x \geq 0, y \geq 0 \end{array} \right\}$$



Corner point

A(20, 0)

B $\left(0, \frac{80}{3}\right)$

C(10, 20)

$$z = 15x + 10y$$

300

$$\frac{800}{3} \approx 266.6$$

350 ← maximum

Maximum profit = ₹ 350

when $x = 10, y = 20$. $\frac{1}{2}$

Correct Figure

If a student has interpreted the language of the question in a different way, then the LPP will be of the type:

$$\text{Maximise profit} \quad z = 15x + 10y$$

$$\text{Subject to} \quad 2x + y \leq 8$$

$$2x + 3y \leq 8$$

$$x \geq 0, y \geq 0$$

This is be accepted and marks may be given accordingly.