NCERT Solution For Class 11 Maths Chapter 15
Statistics

## EXERCISE 15.1

Find the mean deviation about the mean for the data in Exercises 1 and 2.

## 1. 4, 7, 8, 9, 10, 12, 13, 17

## Solution:-

First we have to find ( $x$ ) of the given data

$$
\overline{\mathrm{x}}=\frac{1}{8} \sum_{\mathrm{i}=1}^{8} \mathrm{x}_{\mathrm{i}}=\frac{80}{8}=10
$$

So, the respective values of the deviations from mean,
i.e., $x_{i}-\bar{x}$ are, $10-4=6,10-7=3,10-8=2,10-10=0$,
$10-12=-2,10-13=-3,10-17=-7$
$6,3,2,1,0,-2,-3,-7$
Now absolute values of the deviations, $6,3,2,1,0,2,3,7$
$\therefore \sum_{\mathrm{i}=1}^{8}\left|\mathrm{x}_{\mathrm{i}}-\overline{\mathrm{x}}\right|=24$
$\mathrm{MD}=$ sum of deviations/ number of observations

$$
\begin{aligned}
& =24 / 8 \\
& =3
\end{aligned}
$$

So, the mean deviation for the given data is 3 .

## 2. 38, 70, 48, 40, 42, 55, 63, 46, 54, 44 Solution:-

First we have to find ( $x$ ) of the given data
$\overline{\mathrm{x}}=\frac{1}{10} \sum_{\mathrm{i}=1}^{10} \mathrm{x}_{\mathrm{i}}=\frac{500}{10}=50$
So, the respective values of the deviations from mean,
i.e., $x_{i}-\bar{x}$ are, $50-38=-12,50-70=-20,50-48=2,50-40=10,50-42=8$,
$50-55=-5,50-63=-13,50-46=4,50-54=-4,50-44=6$
$-12,20,-2,-10,-8,5,13,-4,4,-6$
Now absolute values of the deviations,

$$
12,20,2,10,8,5,13,4,4,6
$$

$\therefore \sum_{\mathrm{i}=1}^{10}\left|\mathrm{x}_{\mathrm{i}}-\overline{\mathrm{x}}\right|=84$
$\mathrm{MD}=$ sum of deviations/ number of observations

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$$
\begin{aligned}
& =84 / 10 \\
& =8.4
\end{aligned}
$$

So, the mean deviation for the given data is 8.4.

Find the mean deviation about the median for the data in Exercises 3 and 4.
$3.13,17,16,14,11,13,10,16,11,18,12,17$

## Solution:-

First we have to arrange the given observations into ascending order, 10, $11,11,12,13,13,14,16,16,17,17,18$.
The number of observations is 12
Then,
Median $=\left((12 / 2)^{\text {th }}\right.$ observation $+((12 / 2)+1)^{\text {th }}$ observation $) / 2$
$(12 / 2)^{\text {th }}$ observation $=6^{\text {th }}=13$
$(12 / 2)+1)^{\text {th }}$ observation $=6+1$

$$
=7^{\text {th }}=14
$$

Median $=(13+14) / 2$

$$
=27 / 2
$$

$$
=13.5
$$

So, the absolute values of the respective deviations from the median, i.e., $\left|x_{i}-M\right|$ are

$$
3.5,2.5,2.5,1.5,0.5,0.5,0.5,2.5,2.5,3.5,3.5,4.5
$$

$\therefore \sum_{\mathrm{i}=1}^{12}\left|\mathrm{x}_{\mathrm{i}}-\mathrm{M}\right|=28$
Mean Deviation,
M.D. $(M)=\frac{1}{12} \sum_{i=1}^{12}\left|\mathrm{X}_{\mathrm{i}}-\mathrm{M}\right|$

$$
=(1 / 12) \times 28
$$

$$
=2.33
$$

So, the mean deviation about the median for the given data is 2.33 .

## 4. $36,72,46,42,60,45,53,46,51,49$

## Solution:-

First we have to arrange the given observations into ascending order, 36, $42,45,46,46,49,51,53,60,72$.
The number of observations is 10
Then,

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Median $=\left((10 / 2)^{\text {th }}\right.$ observation $+((10 / 2)+1)^{\text {th }}$ observation $) / 2$
$(10 / 2)^{\text {th }}$ observation $=5^{\text {th }}=46$
$(10 / 2)+1)^{\text {th }}$ observation $=5+1$

$$
=6^{\text {th }}=49
$$

Median $=(46+49) / 2$

$$
95
$$

$$
=47.5
$$

So, the absolute values of the respective deviations from the median, i.e., $\left|x_{i}-M\right|$ are $11.5,5.5,2.5,1.5,1.5,1.5,3.5,5.5,12.5,24.5$
$\therefore \sum_{\mathrm{i}=1}^{10}\left|\mathrm{x}_{\mathrm{i}}-\mathrm{M}\right|=70$
Mean Deviation,
M.D. $(M)=\frac{1}{10} \sum_{i=1}^{10}\left|\mathrm{x}_{\mathrm{i}}-\mathrm{M}\right|$

$$
\begin{aligned}
& =(1 / 10) \times 70 \\
& =7
\end{aligned}
$$

So, the mean deviation about the median for the given data is 7 .

Find the mean deviation about the mean for the data in Exercises 5 and 6.
5.

| $\mathrm{x}_{\mathrm{i}}$ | 5 | 10 | 15 | 20 | 25 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}_{\mathrm{i}}$ | 7 | 4 | 6 | 3 | 5 |

## Solution:-

Let us make the table of the given data and append other columns after calculations.

| $\mathbf{X}_{\mathbf{i}}$ | $\mathbf{f}_{\mathbf{i}}$ | $\mathbf{f}_{\mathbf{i}} \mathbf{x}_{\mathbf{i}}$ | $\left\|\mathbf{x}_{\mathbf{i}}-\overline{\mathbf{x}}\right\|$ | $\mathbf{f}_{\mathbf{i}}\left\|\mathbf{x}_{\mathbf{i}}-\overline{\mathbf{x}}\right\|$ |
| :---: | :---: | :---: | :---: | :---: |
| 5 | 7 | 35 | 9 | 63 |
| 10 | 4 | 40 | 4 | 16 |
| 15 | 6 | 90 | 1 | 6 |
| 20 | 3 | 60 | 6 | 18 |
| 25 | 5 | 125 | 11 | 55 |
|  | 25 | 350 |  | 158 |

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The sum of calculated data,
$N=\sum_{i=1}^{5} f_{i}=25, \sum_{i=1}^{5} f_{i} x_{i}=350$
Now, we have to find $(\bar{x})$ by using the formula
$\Rightarrow \overline{\mathrm{x}}=\frac{1}{\mathrm{~N}} \sum_{\mathrm{i}=1}^{5} \mathrm{f}_{\mathrm{i}} \mathrm{x}_{\mathrm{i}}=\frac{1}{25} \times 350=14$
The absolute values of the deviations from the mean, i.e., $\left|x_{i}-x\right|$, as shown in the table. From the table, $\sum_{\mathrm{i}=1}^{5} \mathrm{f}_{\mathrm{i}}\left|\mathrm{x}_{\mathrm{i}}-\overline{\mathrm{x}}\right|=158$

Therefore M.D. $(\bar{x})=\frac{1}{N} \sum_{i=1}^{5} f_{i}\left|x_{i}-\bar{x}\right|$

$$
\begin{aligned}
& =(1 / 25) \times 158 \\
& =6.32
\end{aligned}
$$

So, the mean deviation about the mean for the given data is 6.32 .
6.

| $\mathbf{x i}_{\mathbf{i}}$ | 10 | 30 | 50 | 70 | 90 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}_{\mathrm{i}}$ | 4 | 24 | 28 | 16 | 8 |

## Solution:-

Let us make the table of the given data and append other columns after calculations.

| $\mathbf{X}_{\mathbf{i}}$ | $\mathbf{f}_{\mathbf{i}}$ | $\mathbf{f}_{\mathbf{i}} \mathbf{x}_{\mathbf{i}}$ | $\left\|\mathbf{x}_{\mathbf{i}}-\overline{\mathbf{x}}\right\|$ | $\mathbf{f}_{\mathbf{i}} \mid \mathbf{x}_{\mathbf{i}}-\overline{\mathbf{x}} \mathbf{\|}$ |
| :---: | :---: | :---: | :---: | :---: |
| 10 | 4 | 40 | 40 | 160 |
| 30 | 24 | 720 | 20 | 480 |
| 50 | 28 | 1400 | 0 | 0 |
| 70 | 16 | 1120 | 20 | 320 |
| 90 | 8 | 720 | 40 | 320 |
|  | 80 | 4000 |  | 1280 |

The sum of calculated data,

$$
N=\sum_{i=1}^{5} f_{i}=80, \sum_{i=1}^{5} f_{i} x_{i}=4000
$$

Now, we have to find ( $\overline{\mathrm{x}}$ ) by using the formula
$\Rightarrow \overline{\mathrm{x}}=\frac{1}{\mathrm{~N}} \sum_{\mathrm{i}=1}^{5} \mathrm{f}_{\mathrm{i}} \mathrm{X}_{\mathrm{i}}=\frac{1}{80} \times 4000=50$
The absolute values of the deviations from the mean, i.e., $\left|x_{i}-\bar{x}\right|$, as shown in the table.

From the table,$\sum_{i=1}^{5} \mathrm{f}_{\mathrm{i}}\left|\mathrm{x}_{\mathrm{i}}-\overline{\mathrm{x}}\right|=1280$
Therefore M.D. $(\bar{x})=\frac{1}{N} \sum_{i=1}^{5} f_{i}\left|x_{i}-\bar{x}\right|$

$$
\begin{aligned}
& =(1 / 80) \times 1280 \\
& =16
\end{aligned}
$$

So, the mean deviation about the mean for the given data is 16 .

Find the mean deviation about the median for the data in Exercises 7 and 8.
7.

| $\mathrm{x}_{\mathrm{i}}$ | 5 | 7 | 9 | 10 | 12 | 15 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}_{\mathrm{i}}$ | 8 | 6 | 2 | 2 | 2 | 6 |

## Solution:-

Let us make the table of the given data and append other columns after calculations.

| $\mathbf{X}_{\mathbf{i}}$ | $\mathbf{f}_{\mathbf{i}}$ | c.f. | $\mid \mathbf{x}_{\mathbf{i}}-\mathbf{M} \mathbf{\|}$ | $\mathbf{f}_{\mathbf{i}}\left\|\mathbf{x}_{\mathbf{i}}-\mathbf{M}\right\|$ |
| :---: | :---: | :---: | :---: | :---: |
| 5 | 8 | 8 | 2 | 16 |
| 7 | 6 | 14 | 0 | 0 |

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| 9 | 2 | 16 | 2 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| 10 | 2 | 18 | 3 | 6 |
| 12 | 2 | 20 | 5 | 10 |
| 15 | 6 | 26 | 8 | 48 |

Now, $N=26$, which is even.
Median is the mean of the $13^{\text {th }}$ and $14^{\text {th }}$ observations. Both of these observations lie in the cumulative frequency 14 , for which the corresponding observation is 7 .
Then,
Median $=\left(13^{\text {th }}\right.$ observation $+14^{\text {th }}$ observation $) / 2$

$$
\begin{aligned}
& =(7+7) / 2 \\
& =14 / 2 \\
& =7
\end{aligned}
$$

So, the absolute values of the respective deviations from the median, i.e., $\left|x_{i}-M\right|$ are shown in the table.
Therefore $\sum_{i=1}^{6} \mathrm{f}_{\mathrm{i}}=26$ and $\sum_{\mathrm{i}=1}^{6} \mathrm{f}_{\mathrm{i}}\left|\mathrm{x}_{\mathrm{i}}-\mathrm{M}\right|=84$

And
M. D. $(M)=\frac{1}{N} \sum_{i=1}^{6} f_{i}\left|x_{i}-M\right|$

$$
=(1 / 26) \times 84
$$

$$
=3.23
$$

Hence, the mean deviation about the median for the given data is 3.23 .
8.

| $\mathbf{x i}_{\mathbf{i}}$ | 15 | 21 | 27 | 30 | 35 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}_{\mathrm{i}}$ | 3 | 5 | 6 | 7 | 8 |

## Solution:-

Let us make the table of the given data and append other columns after calculations.

| $\mathbf{X i}_{\mathbf{i}}$ | $\mathbf{f}_{\mathbf{i}}$ | c.f. | $\left\|\mathbf{x}_{\mathbf{i}}-\mathbf{M}\right\|$ | $\mathbf{f}_{\mathbf{i}}\left\|\mathbf{x}_{\mathbf{i}}-\mathbf{M}\right\|$ |
| :---: | :---: | :---: | :---: | :---: |

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| 15 | 3 | 3 | 13.5 | 40.5 |
| :---: | :---: | :---: | :---: | :---: |
| 21 | 5 | 8 | 7.5 | 37.5 |
| 27 | 6 | 14 | 1.5 | 9 |
| 30 | 7 | 21 | 1.5 | 10.5 |
| 35 | 8 | 29 | 6.5 | 52 |

Now, $\mathrm{N}=30$, which is even.
Median is the mean of the $15^{\text {th }}$ and $16^{\text {th }}$ observations. Both of these observations lie in the cumulative frequency 21 , for which the corresponding observation is 30 .
Then,
Median $=\left(15^{\text {th }}\right.$ observation $+16^{\text {th }}$ observation $) / 2$

$$
\begin{aligned}
& =(30+30) / 2 \\
& =60 / 2 \\
& =30
\end{aligned}
$$

So, the absolute values of the respective deviations from the median, i.e., $\left|x_{i}-M\right|$ are shown in the table.
Therefore $\sum_{\mathrm{i}=1}^{5} \mathrm{f}_{\mathrm{i}}=29$ and $\sum_{\mathrm{i}=1}^{5} \mathrm{f}_{\mathrm{i}}\left|\mathrm{x}_{\mathrm{i}}-\mathrm{M}\right|=149.5$
And
M. D. $(M)=\frac{1}{N} \sum_{i=1}^{6} f_{i}\left|x_{i}-M\right|$

$$
\begin{aligned}
& =(1 / 29) \times 149.5 \\
& =5.1
\end{aligned}
$$

Hence, the mean deviation about the median for the given data is 5.1.

Find the mean deviation about the mean for the data in Exercises 9 and 10.
9.

| Income <br> per day <br> in 囵 | $0-100$ | $100-$ <br> 200 | $200-$ <br> 300 | $300-$ <br> 400 | $400-$ <br> 500 | $500-$ <br> 600 | $600-$ <br> 700 | $700-$ <br> 800 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number <br> of <br> persons | 4 | 8 | 9 | 10 | 7 | 5 | 4 | 3 |

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Solution:-

Let us make the table of the given data and append other columns after calculations.

| Income per <br> day in $0^{2}$ | Number of <br> persons $\mathbf{f}_{\mathbf{i}}$ | Mid- points <br> $\mathbf{x}_{\mathbf{i}}$ | $\mathbf{f}_{\mathbf{i} \mathbf{x}_{\mathbf{i}}}$ | $\mid \mathbf{x}_{\mathbf{i}}-\overline{\mathbf{x}} \mathbf{\|}$ | $\mathbf{f}_{\mathbf{i}} \mid \mathbf{x}_{\mathbf{i}}-\overline{\mathbf{x}} \mathbf{\|}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $0-100$ | 4 | 50 | 200 | 308 | 1232 |
| $100-200$ | 8 | 150 | 1200 | 208 | 1664 |
| $200-300$ | 9 | 250 | 2250 | 108 | 972 |
| $300-400$ | 10 | 350 | 3500 | 8 | 80 |
| $400-500$ | 7 | 450 | 3150 | 92 | 644 |
| $500-600$ | 5 | 550 | 2750 | 192 | 960 |
| $600-700$ | 4 | 650 | 2600 | 292 | 1160 |
| $700-800$ | 3 | 750 | 2250 | 392 | 1176 |
|  | 50 |  | 17900 |  | 7896 |

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The sum of calculated data,
$N=\sum_{i=1}^{8} f_{i}=50, \sum_{i=1}^{8} f_{i} \mathrm{X}_{\mathrm{i}}=17900$
Now, we have to find $(\bar{x})$ by using the formula
$\Rightarrow \overline{\mathrm{x}}=\frac{1}{\mathrm{~N}} \sum_{\mathrm{i}=1}^{8} \mathrm{f}_{\mathrm{i}} \mathrm{X}_{\mathrm{i}}=\frac{1}{50} \times 17900=358$
The absolute values of the deviations from the mean, i.e., $\left|x_{i}-\bar{x}\right|$, as shown in the table.

So, $\sum_{\mathrm{i}=1}^{8} \mathrm{f}_{\mathrm{i}}\left|\mathrm{x}_{\mathrm{i}}-\overline{\mathrm{x}}\right|=7896$
And

$$
\text { M.D. } \begin{aligned}
(\overline{\mathrm{x}}) & =\frac{1}{N} \sum_{\mathrm{i}=1}^{8} \mathrm{f}_{\mathrm{i}}\left|\mathrm{x}_{\mathrm{i}}-\overline{\mathrm{x}}\right| \\
= & (1 / 50) \times 7896 \\
= & 157.92
\end{aligned}
$$

Hence, the mean deviation about the mean for the given data is 157.92 .
10.

| Height <br> in cms | $95-$ <br> 105 | $105-$ <br> 115 | $115-$ <br> 125 | $125-$ <br> 135 | $135-$ <br> 145 | $145-$ <br> 155 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number <br> of boys | 9 | 13 | 26 | 30 | 12 | 10 |

## Solution:-

Let us make the table of the given data and append other columns after calculations.

| Height in <br> cms | Number of <br> boys $\mathbf{f}_{\mathbf{i}}$ | Mid-points <br> $\mathbf{x}_{\mathbf{i}}$ | $\mathbf{f}_{\mathbf{i}} \mathbf{x}_{\mathbf{i}}$ | $\left\|\mathbf{x}_{\mathbf{i}}-\overline{\mathbf{x}}\right\|$ | $\mathbf{f}_{\mathbf{i}}\left\|\mathbf{x}_{\mathbf{i}}-\overline{\mathbf{x}}\right\|$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $95-105$ | 9 | 100 | 900 | 25.3 | 227.7 |
| $105-115$ | 13 | 110 | 1430 | 15.3 | 198.9 |
| $115-125$ | 26 | 120 | 3120 | 5.3 | 137.8 |

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| $125-135$ | 30 | 130 | 3900 | 4.7 | 141 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $135-145$ | 12 | 140 | 1680 | 14.7 | 176.4 |
| $145-155$ | 10 | 150 | 1500 | 24.7 | 247 |
|  | 100 |  | 12530 |  | 1128.8 |

The sum of calculated data,
$N=\sum_{i=1}^{6} f_{i}=100, \sum_{i=1}^{6} f_{i} x_{i}=12530$
Now, we have to find ( $\overline{\mathrm{x}}$ ) by using the formula
$\Rightarrow \overline{\mathrm{x}}=\frac{1}{\mathrm{~N}} \sum_{\mathrm{i}=1}^{6} \mathrm{f}_{\mathrm{i}} \mathrm{x}_{\mathrm{i}}=\frac{1}{100} \times 12530=125.3$
The absolute values of the deviations from the mean, i.e., $\left|x_{i}-\bar{x}\right|$, as shown in the table.

So $\sum_{i=1}^{6} \mathrm{f}_{\mathrm{i}}\left|\mathrm{x}_{\mathrm{i}}-\overline{\mathrm{x}}\right|=1128.8$
And M.D. $(\bar{x})=\frac{1}{N} \sum_{i=1}^{6} f_{i}\left|x_{i}-\bar{x}\right|$

$$
\begin{aligned}
& =(1 / 100) \times 1128.8 \\
& =11.28
\end{aligned}
$$

Hence, the mean deviation about the mean for the given data is 11.28 .
11. Find the mean deviation about median for the following data:

| Marks | $0-10$ | $10-20$ | $20-30$ | $30-40$ | $40-50$ | $50-60$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number <br> of girls | 6 | 8 | 14 | 16 | 4 | 2 |

Solution:-
Let us make the table of the given data and append other columns after calculations.

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| Marks | Number of <br> Girls $\mathbf{f}_{\mathbf{i}}$ | Cumulative <br> frequency <br> (c.f.) | Mid-points <br> $\mathbf{x}_{\mathbf{i}}$ | $\mid \mathbf{x}_{\mathbf{i}}-$ Med $\mid$ | $\mathbf{f}_{\mathbf{i}} \mid \mathbf{x}_{\mathbf{i}}-$ Med $\mid$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $0-10$ | 6 | 6 | 5 | 22.85 | 137.1 |
| $10-20$ | 8 | 14 | 15 | 12.85 | 102.8 |
| $20-30$ | 14 | 28 | 25 | 2.85 | 39.9 |
| $30-40$ | 16 | 44 | 35 | 7.15 | 114.4 |
| $40-50$ | 4 | 48 | 45 | 17.15 | 68.6 |
| $50-60$ | 2 | 50 | 55 | 27.15 | 54.3 |
|  | 50 |  |  |  | 517.1 |

The class interval containing $\mathrm{N}^{\text {th }} / 2$ or $25^{\text {th }}$ item is $20-30$ So, $20-30$ is the median class.
Then,
Median $=1+(((N / 2)-c) / f) \times h$
Where, $\mathrm{l}=20, \mathrm{c}=14, \mathrm{f}=14, \mathrm{~h}=10$ and $\mathrm{n}=50$
Median $=20+(((25-14)) / 14) \times 10$

$$
\begin{aligned}
& =20+7.85 \\
& =27.85
\end{aligned}
$$

The absolute values of the deviations from the median, i.e., $\mid \mathrm{x}_{\mathrm{i}}$ - Med $\mid$, as shown in the table.

So $\sum_{i=1}^{6} \mathrm{f}_{\mathrm{i}} \mid \mathrm{x}_{\mathrm{i}}-$ Med. $\mid=517.1$
And M.D. (M) $=\frac{1}{N} \sum_{i=1}^{6} f_{i}\left|x_{i}-M e d.\right|$

$$
\begin{aligned}
& =(1 / 50) \times 517.1 \\
& =10.34
\end{aligned}
$$

Hence, the mean deviation about the median for the given data is 10.34 .
12. Calculate the mean deviation about median age for the age distribution of 100 persons given below:

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| Age <br> (in <br> years) | $16-20$ | $21-25$ | $26-30$ | $31-35$ | $36-40$ | $41-45$ | $46-50$ | $51-55$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number | 5 | 6 | 12 | 14 | 26 | 12 | 16 | 9 |

[Hint Convert the given data into continuous frequency distribution by subtracting 0.5 from the lower limit and adding 0.5 to the upper limit of each class interval] Solution:The given data is converted into continuous frequency distribution by subtracting 0.5 from the lower limit and adding the 0.5 to the upper limit of each class intervals and append other columns after calculations.

| Age | Number $\mathbf{f}_{\mathbf{i}}$ | Cumulative <br> frequency <br> (c.f.) | Mid- points <br> $\mathbf{x}_{\mathbf{i}}$ | $\mid \mathbf{x}_{\mathbf{i}}-$ Med $\mid$ | $\mathbf{f}_{\mathbf{i}} \mid \mathbf{x}_{\mathbf{i}}-$ Med $\mid$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $15.5-20.5$ | 5 | 5 | 18 | 20 | 100 |
| $20.5-25.5$ | 6 | 11 | 23 | 15 | 90 |
| $25.5-30.5$ | 12 | 23 | 28 | 10 | 120 |
| $30.5-35.5$ | 14 | 37 | 33 | 5 | 70 |
| $35.5-40.5$ | 26 | 63 | 38 | 0 | 0 |
| $40.5-45.5$ | 12 | 75 | 43 | 5 | 60 |
| $45.5-50.5$ | 16 | 91 | 48 | 10 | 160 |
| $50.5-55.5$ | 9 | 100 | 53 | 15 | 135 |
|  | 100 |  |  |  | 735 |

The class interval containing $\mathrm{N}^{\text {th }} / 2$ or $50^{\text {th }}$ item is $35.5-40.5$ So, $35.5-40.5$ is the median class.
Then,
Median $=1+(((N / 2)-c) / f) \times h$
Where, $I=35.5, c=37, f=26, h=5$ and $N=100$
Median $=35.5+(((50-37)) / 26) \times 5$

$$
\begin{aligned}
& =35.5+2.5 \\
& =38
\end{aligned}
$$

The absolute values of the deviations from the median, i.e., $\left|x_{i}-M e d\right|$, as shown in the table.

$$
\left.\begin{aligned}
& \text { So } \sum_{i=1}^{8} \mathrm{f}_{\mathrm{i}} \mid \mathrm{x}_{\mathrm{i}}-\text { Med. } \mid=735 \\
& \text { And M.D. }(\mathrm{M})
\end{aligned}=\frac{1}{\mathrm{~N}} \sum_{\mathrm{i}=1}^{6} \mathrm{f}_{\mathrm{i}} \right\rvert\, \mathrm{x}_{\mathrm{i}} \text {-Med. } \mid \text { } \quad \begin{aligned}
& (1 / 100) \times 735 \\
& =7.35
\end{aligned}
$$

Hence, the mean deviation about the median for the given data is 7.35 .

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## EXERCISE 15.2

Find the mean and variance for each of the data in Exercise 1 to 5.

1. $6,7,10,12,13,4,8,12$

Solution:-
We have,
Mean $=\overline{\mathrm{x}}=\frac{\sum_{\mathrm{i}=1}^{\mathrm{a}} \mathrm{x}_{\mathrm{i}}}{\mathrm{n}}$
Where, $\mathrm{n}=$ number of observation

$$
\sum_{\mathrm{i}=1}^{\mathrm{a}} \mathrm{x}_{\mathrm{i}}=\text { sum of total observation }
$$

So, $x=(6+7+10+12-13+4+8+12) / 8$

$$
\begin{aligned}
& =72 / 8 \\
& =9
\end{aligned}
$$

Let us make the table of the given data and append other columns after calculations.

| $\mathbf{x}_{\mathbf{i}}$ | Deviations from mean <br> $\left(\mathbf{x}_{\mathbf{i}}-\overline{\mathbf{x}}\right)$ | $\left(\mathbf{x}_{\mathbf{i}}-\overline{\mathbf{x}}\right)^{\mathbf{2}}$ |
| :---: | :---: | :---: |
| 6 | $6-9=-3$ | 9 |
| 7 | $7-9=-2$ | 4 |
| 10 | $10-9=1$ | 1 |
| 12 | $12-9=3$ | 9 |
| 13 | $13-9=4$ | 16 |
| 4 | $4-9=-5$ | 25 |
| 8 | $8-9=-1$ | 1 |
| 12 | $12-9=3$ | 9 |
|  |  | 74 |

We know that Variance,
$\sigma^{2}=\frac{1}{n} \sum_{i=1}^{a}\left(\mathrm{x}_{\mathrm{i}}-\overline{\mathrm{x}}\right)^{2}$
$\sigma^{2}=(1 / 8) \times 74$

$$
=9.2
$$

$\therefore$ Mean $=9$ and Variance $=9.25$
2. First n natural numbers

## NCERT Solution For Class 11 Maths Chapter 15

Statistics

## Solution:-

We know that Mean = Sum of all observations/Number of observations
$\therefore$ Mean, $\bar{x}=((n(n+1)) 2) / n \quad=(n+1) / 2$
and also WKT Variance,

$$
\sigma^{2}=\frac{1}{n} \sum_{\mathrm{i}=1}^{\mathrm{a}}\left(\mathrm{x}_{\mathrm{i}}-\overline{\mathrm{x}}\right)^{2}
$$

By substitute that value of $\bar{x}$ we get,

$$
=\frac{1}{n} \sum_{i=1}^{n}\left(x_{i}-\frac{n+1}{2}\right)^{2}
$$

We know that $(a-b)^{2}=a^{2}-2 a b+b^{2}$

$$
=\frac{1}{\mathrm{n}} \sum_{\mathrm{i}=1}^{\mathrm{n}}\left(\mathrm{x}_{\mathrm{i}}\right)^{2}-\frac{1}{\mathrm{n}} \sum_{\mathrm{i}=1}^{\mathrm{n}} 2 \mathrm{x}_{\mathrm{i}}\left(\frac{\mathrm{n}+1}{2}\right)+\frac{1}{\mathrm{n}} \sum_{\mathrm{i}=1}^{\mathrm{n}}\left(\frac{\mathrm{n}+1}{2}\right)^{2}
$$

Substituting the summation values

$$
=\frac{1}{\mathrm{n}} \frac{\mathrm{n}(\mathrm{n}+1)(2 \mathrm{n}+1)}{6}-\frac{\mathrm{n}+1}{\mathrm{n}}\left[\frac{\mathrm{n}(\mathrm{n}+1)}{2}\right]+\frac{(\mathrm{n}+1)^{2}}{4 \mathrm{n}} \times \mathrm{n}
$$

Multiplying and Computing

$$
=\frac{(\mathrm{n}+1)(2 \mathrm{n}+1)}{6}-\frac{(\mathrm{n}+1)^{2}}{2}+\frac{(\mathrm{n}+1)^{2}}{4}
$$

By taking LCM and simplifying, we get

$$
=\frac{(n+1)(2 n+1)}{6}-\frac{(n+1)^{2}}{4}
$$

By taking ( $n+1$ ) common from each term, we get

$$
\begin{aligned}
& =(n+1)\left[\frac{4 n+2-3 n-3}{12}\right] \\
& =\frac{(n+1)(n-1)}{12}
\end{aligned}
$$

WKT, $(a+b)(a-b)=a^{2}-b^{2} \sigma^{2}$

$$
=\left(n^{2}-1\right) / 12
$$

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$\therefore$ Mean $=(n+1) / 2$ and Variance $=\left(n^{2}-1\right) / 12$

## 3. First 10 multiples of 3

## Solution:-

First we have to write the first 10 multiples of 3,
$3,6,9,12,15,18,21,24,27,30$
We have,
Mean $=\overline{\mathrm{x}}=\frac{\sum_{\mathrm{i}=1}^{\mathrm{a}} \mathrm{x}_{\mathrm{i}}}{\mathrm{n}}$
Where, $\mathrm{n}=$ number of observation

$$
\sum_{\mathrm{i}=1}^{\mathrm{a}} \mathrm{x}_{\mathrm{i}}=\text { sum of total observation }
$$

So, $\bar{x}=(3+6+9+12+15+18+21+24+27+30) / 10$
= 165/10
$=16.5$
Let us make the table of the data and append other columns after calculations.

| $\mathbf{x}_{\mathbf{i}}$ | Deviations from mean <br> $\left(\mathbf{x}_{\mathbf{i}}-\overline{\mathbf{x}}\right)$ | $\left.\mathbf{( x}_{\mathbf{i}}-\overline{\mathbf{x}}\right)^{\mathbf{2}}$ |
| :---: | :---: | :---: |
| 3 | $3-16.5=-13.5$ | 182.25 |
| 6 | $6-16.5=-10.5$ | 110.25 |
| 9 | $9-16.5=-7.5$ | 56.25 |
| 12 | $12-16.5=-4.5$ | 20.25 |
| 15 | $15-16.5=-1.5$ | 2.25 |
| 18 | $18-16.5=1.5$ | 2.25 |
| 21 | $21-16.5=-4.5$ | 20.25 |
| 24 | $24-16.5=7.5$ | 56.25 |
| 27 | $27-16.5=10.5$ | 110.25 |
| 30 | $30-16.5=13.5$ | 182.25 |
|  |  | 742.5 |

Then, Variance

$$
\sigma^{2}=\frac{1}{n} \sum_{i=1}^{\mathrm{a}}\left(\mathrm{x}_{\mathrm{i}}-\overline{\mathrm{x}}\right)^{2}
$$

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$$
\begin{aligned}
& =(1 / 10) \times 742.5 \\
& =74.25
\end{aligned}
$$

$\therefore$ Mean $=16.5$ and Variance $=74.25$
4.

| $\mathrm{xi}_{\mathrm{i}}$ | 6 | 10 | 14 | 18 | 24 | 28 | 30 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}_{\mathrm{i}}$ | 2 | 4 | 7 | 12 | 8 | 4 | 3 |

Solution:-

Let us make the table of the given data and append other columns after calculations.

| $\mathbf{X}_{\mathbf{i}}$ | $\mathbf{f}_{\mathbf{i}}$ | $\mathbf{f i x}_{\mathbf{i}}$ | Deviations from mean <br> $\left(\mathbf{x}_{\mathbf{i}}-\overline{\mathbf{x}}\right)$ | $\left(\mathbf{x}_{\mathbf{i}}-\overline{\mathbf{x}}\right)^{\mathbf{2}}$ | $\mathbf{f}_{\mathbf{i}}\left(\mathbf{x}_{\mathbf{i}}-\overline{\mathbf{x}}\right)^{\mathbf{2}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 6 | 2 | 12 | $6-19=13$ | 169 | 338 |
| 10 | 4 | 40 | $10-19=-9$ | 81 | 324 |
| 14 | 7 | 98 | $14-19=-5$ | 25 | 175 |
| 18 | 12 | 216 | $18-19=-1$ | 1 | 12 |
| 24 | 8 | 192 | $24-19=5$ | 25 | 200 |
| 28 | 4 | 112 | $28-19=9$ | 81 | 324 |
| 30 | 3 | 90 | $30-19=11$ | 121 | 363 |
|  | $\mathrm{~N}=40$ | 760 |  |  | 1736 |

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Then Mean, $\overline{\mathrm{x}}=\frac{\sum_{\mathrm{i}=1}^{\mathrm{a}} \mathrm{f}_{\mathrm{i}} \mathrm{x}_{\mathrm{i}}}{\mathrm{N}}$
Where $\mathrm{N}=\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{f}_{\mathrm{i}}$

$$
\begin{aligned}
\bar{x} & =760 / 40 \\
& =19
\end{aligned}
$$

Now, Variance, $\sigma^{2}=\frac{1}{N} \sum_{\mathrm{i}=1}^{\mathrm{a}} \mathrm{f}_{\mathrm{i}}\left(\mathrm{x}_{\mathrm{i}}-\overline{\mathrm{x}}\right)^{2}$

$$
\begin{aligned}
& =(1 / 40) \times 1736 \\
& =43.4
\end{aligned}
$$

$\therefore$ Mean $=19$ and Variance $=43.4$
5.

| $\mathrm{xi}_{\mathrm{i}}$ | 92 | 93 | 97 | 98 | 102 | 104 | 109 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}_{\mathrm{i}}$ | 3 | 2 | 3 | 2 | 6 | 3 | 3 |

## Solution:-

Let us make the table of the given data and append other columns after calculations.

| $\mathbf{X}_{\mathbf{i}}$ | $\mathbf{f}_{\mathbf{i}}$ | $\mathbf{f}_{\mathbf{i} \mathbf{x}_{\mathbf{i}}}$ | Deviations from mean <br> $\left(\mathbf{x}_{\mathbf{i}}-\overline{\mathbf{x}}\right)$ | $\left(\mathbf{x}_{\mathbf{i}}-\overline{\mathbf{x}}\right)^{\mathbf{2}}$ | $\mathbf{f}_{\mathbf{i}}\left(\mathbf{x}_{\mathbf{i}}-\overline{\mathbf{x}}\right)^{\mathbf{2}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 92 | 3 | 276 | $92-100=-8$ | 64 | 192 |
| 93 | 2 | 186 | $93-100=-7$ | 49 | 98 |
| 97 | 3 | 291 | $97-100=-3$ | 9 | 27 |
| 98 | 2 | 196 | $98-100=-2$ | 4 | 8 |
| 102 | 6 | 612 | $102-100=2$ | 4 | 24 |
| 104 | 3 | 312 | $104-100=4$ | 16 | 48 |
| 109 | 3 | 327 | $109-100=9$ | 81 | 243 |
|  | $\mathrm{~N}=22$ | 2200 |  |  | 640 |

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Then Mean, $\overline{\mathrm{x}}=\frac{\sum_{\mathrm{i}=1}^{\mathrm{a}} \mathrm{f}_{\mathrm{i}} \mathrm{x}_{\mathrm{i}}}{\mathrm{N}}$
Where $\mathrm{N}=\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{f}_{\mathrm{i}}$
$\overline{\mathrm{x}}=2200 / 22$
$=100$
Now, Variance, $\sigma^{2}=\frac{1}{\mathrm{~N}} \sum_{\mathrm{i}=1}^{\mathrm{a}} \mathrm{f}_{\mathrm{i}}\left(\mathrm{x}_{\mathrm{i}}-\overline{\mathrm{x}}\right)^{2}$

$$
\begin{aligned}
& =(1 / 22) \times 640 \\
& =29.09
\end{aligned}
$$

$\therefore$ Mean $=100$ and Variance $=29.09$
6. Find the mean and standard deviation using short-cut method.

| $\mathrm{x}_{\mathrm{i}}$ | 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}_{\mathrm{i}}$ | 2 | 1 | 12 | 29 | 25 | 12 | 10 | 4 | 5 |

## Solution:-

Let the assumed mean $A=64$. Here $h=1$
We obtain the following table from the given data.

| $\mathbf{X}_{\mathbf{i}}$ | Frequency <br> $\mathbf{f}_{\mathbf{i}}$ | $\mathbf{Y}_{\mathbf{i}}=\left(\mathbf{x}_{\mathbf{i}}-\mathbf{A}\right) / \mathbf{h}$ | $\mathbf{Y}_{\mathbf{i} \mathbf{2}}$ | $\mathbf{f}_{\mathbf{i} \mathbf{y}_{\mathbf{i}}}$ | $\mathbf{f}_{\mathbf{i} \mathbf{y}_{\mathbf{i}}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 60 | 2 | -4 | 16 | -8 | 32 |
| 61 | 1 | -3 | 9 | -3 | 9 |
| 62 | 12 | -2 | 4 | -24 | 48 |
| 63 | 29 | -1 | 1 | -29 | 29 |
| 64 | 25 | 0 | 0 | 0 | 0 |
| 65 | 12 | 1 | 1 | 12 | 12 |
| 66 | 10 | 2 | 4 | 20 | 40 |
| 67 | 4 | 3 | 9 | 12 | 36 |
| 68 | 5 | 4 | 16 | 20 | 80 |
|  |  |  |  | 0 | 286 |

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Mean,
$\bar{x}=A+\frac{\sum_{i=1}^{a} f_{i} y_{i}}{N} \times h$
Where $A=64, h=1$
So, $x \equiv 64+((0 / 100) \times 1)$
$=64+0$
$=64$
Then, variance,

$$
\begin{aligned}
\sigma^{2} & =\frac{h^{2}}{N^{2}}\left[N \Sigma f_{i} y_{i}^{2}-\left(\Sigma f_{i} y_{i}\right)^{2}\right] \\
\sigma^{2} & =\left(1^{2} / 100^{2}\right)\left[100(286)-0^{2}\right] \\
& =(1 / 10000)[28600-0] \\
& =28600 / 10000 \\
& =2.86
\end{aligned}
$$

Hence, standard deviation $=\sigma=\sqrt{ } 2.886$

$$
=1.691
$$

$\therefore$ Mean $=64$ and Standard Deviation $=1.691$

Find the mean and variance for the following frequency distributions in Exercises 7 and 8.
7.

| Classes | $0-30$ | $30-60$ | $60-90$ | $90-120$ | $120-150$ | $150-180$ | $180-210$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequencies | 2 | 3 | 5 | 10 | 3 | 5 | 2 |

## Solution:-

Let us make the table of the given data and append other columns after calculations.

| Classes | Frequency <br> $\mathbf{f i}_{\mathbf{i}}$ | Mid- <br> points <br> $\mathbf{x i}$ | $\mathbf{f i x}_{\mathbf{i}}$ | $\left(\mathbf{x}_{\mathbf{i}}-\overline{\mathbf{x}}\right)$ | $\left(\mathbf{x}_{\mathbf{i}}-\overline{\mathbf{x}}\right)^{\mathbf{2}}$ | $\mathbf{f}_{\mathbf{i}}\left(\mathbf{x}_{\mathbf{i}}-\overline{\mathbf{x}}\right)^{\mathbf{2}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $0-30$ | 2 | 15 | 30 | -92 | 8464 | 16928 |
| $30-60$ | 3 | 45 | 135 | -62 | 3844 | 11532 |
| $60-90$ | 5 | 75 | 375 | -32 | 1024 | 5120 |
| $90-120$ | 10 | 105 | 1050 | -2 | 4 | 40 |

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| $120-150$ | 3 | 135 | 405 | 28 | 784 | 2352 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $150-180$ | 5 | 165 | 825 | 58 | 3364 | 16820 |
| $180-210$ | 2 | 195 | 390 | 88 | 7744 | 15488 |
|  | $\mathrm{~N}=30$ |  | 3210 |  |  | 68280 |

Then Mean, $\overline{\mathrm{x}}=\frac{\sum_{\mathrm{i}=1}^{\mathrm{a}} \mathrm{f}_{\mathrm{i}} \mathrm{x}_{\mathrm{i}}}{\mathrm{N}}$
Where $\mathrm{N}=\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{f}_{\mathrm{i}}$
$\overline{\mathrm{x}}=3210 / 30$
$=107$
Now, Variance, $\sigma^{2}=\frac{1}{N} \sum_{i=1}^{a} \mathrm{f}_{\mathrm{i}}\left(\mathrm{x}_{\mathrm{i}}-\overline{\mathrm{x}}\right)^{2}$

$$
\begin{aligned}
& =(1 / 30) \times 68280 \\
& =2276
\end{aligned}
$$

$\therefore$ Mean $=107$ and Variance $=2276$
8.

| Classes | $0-10$ | $10-20$ | $20-30$ | $30-40$ | $40-50$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Frequencies | 5 | 8 | 15 | 16 | 6 |

## Solution:-

Let us make the table of the given data and append other columns after calculations.

| Classes | Frequency <br> $\mathbf{f}_{\mathbf{i}}$ | Mid- <br> points <br> $\mathbf{x}_{\mathbf{i}}$ | $\mathbf{f i x}_{\mathbf{i}}$ | $\left(\mathbf{x}_{\mathbf{i}}-\overline{\mathbf{x}}\right)$ | $\left(\mathbf{x}_{\mathbf{i}}-\overline{\mathbf{x}}\right)^{\mathbf{2}}$ | $\mathbf{f}_{\mathbf{i}}\left(\mathbf{x}_{\mathbf{i}}-\overline{\mathbf{x}}\right)^{\mathbf{2}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $0-10$ | 5 | 5 | 25 | -22 | 484 | 2420 |
| $10-20$ | 8 | 15 | 120 | -12 | 144 | 1152 |
| $20-30$ | 15 | 25 | 375 | -2 | 4 | 60 |

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| $30-40$ | 16 | 35 | 560 | 8 | 64 | 1024 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $40-50$ | 6 | 45 | 270 | 18 | 324 | 1944 |
|  | $\mathrm{~N}=50$ |  | 1350 |  |  | 6600 |

Then Mean, $\overline{\mathrm{x}}=\frac{\sum_{\mathrm{i}=1}^{\mathrm{a}} \mathrm{f}_{\mathrm{i}} \mathrm{x}_{\mathrm{i}}}{\mathrm{N}}$
Where $\mathrm{N}=\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{f}_{\mathrm{i}}$
$\overline{\mathrm{x}}=1350 / 50$
$=27$

Now, Variance, $\sigma^{2}=\frac{1}{\mathrm{~N}} \sum_{\mathrm{i}=1}^{\mathrm{a}} \mathrm{f}_{\mathrm{i}}\left(\mathrm{x}_{\mathrm{i}}-\overline{\mathrm{x}}\right)^{2}$

$$
\begin{aligned}
& =(1 / 50) \times 6600 \\
& =132
\end{aligned}
$$

$\therefore$ Mean $=27$ and Variance $=132$
9. Find the mean, variance and standard deviation using short-cut method

| Height in <br> cms | $70-$ <br> 75 | $75-$ <br> 80 | $80-$ <br> 85 | $85-$ <br> 90 | $90-$ <br> 95 | $95-$ <br> 100 | $100-$ <br> 105 | $105-$ <br> 110 | $110-$ <br> 115 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequencies | 3 | 4 | 7 | 7 | 15 | 9 | 6 | 6 | 3 |

## Solution:-

Let the assumed mean, $\mathrm{A}=92.5$ and $\mathrm{h}=5$
Let us make the table of the given data and append other columns after calculations.

| Height (class) | Number of <br> children <br> Frequency $\mathbf{f}_{\mathbf{i}}$ | Midpoint <br> $\mathbf{X}_{\mathbf{i}}$ | $\mathbf{Y}_{\mathbf{i}}=\left(\mathbf{x}_{\mathbf{i}}-\right.$ <br> $\mathbf{A}) / \mathbf{h}$ | $\mathbf{Y}_{\mathbf{i} 2}$ | $\mathbf{f}_{\mathbf{i}} \mathbf{i}$ | $\mathbf{f i y i z}^{\mathbf{2}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $70-75$ | 2 | 72.5 | -4 | 16 | -12 | 48 |
| $75-80$ | 1 | 77.5 | -3 | 9 | -12 | 36 |
| $80-85$ | 12 | 82.5 | -2 | 4 | -14 | 28 |
| $85-90$ | 29 | 87.5 | -1 | 1 | -7 | 7 |
| $90-95$ | 25 | 92.5 | 0 | 0 | 0 | 0 |

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| $95-100$ | 12 | 97.5 | 1 | 1 | 9 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $100-105$ | 10 | 102.5 | 2 | 4 | 12 | 24 |
| $105-110$ | 4 | 107.5 | 3 | 9 | 18 | 54 |
| $110-115$ | 5 | 112.5 | 4 | 16 | 12 | 48 |
|  | $\mathrm{~N}=60$ |  |  |  | 6 | 254 |

Mean,
$\overline{\mathrm{x}}=\mathrm{A}+\frac{\sum_{\mathrm{i}=1}^{\mathrm{a}} \mathrm{f}_{\mathrm{i}} \mathrm{y}_{\mathrm{i}}}{\mathrm{N}} \times \mathrm{h}$
Where, $A=92.5, h=5$
So, $\bar{x}=92.5+((6 / 60) \times 5)$

$$
\begin{aligned}
& =92.5+1 / 2 \\
& =92.5+0.5 \\
& =93
\end{aligned}
$$

Then, Variance,

$$
\begin{aligned}
\sigma^{2} & =\frac{\mathrm{h}^{2}}{\mathrm{~N}^{2}}\left[\mathrm{~N} \Sigma \mathrm{f}_{\mathrm{i}} \mathrm{y}_{\mathrm{i}}^{2}-\left(\Sigma \mathrm{f}_{\mathrm{i}} \mathrm{y}_{\mathrm{i}}\right)^{2}\right] \\
\sigma^{2} & =\left(5^{2} / 60^{2}\right)\left[60(254)-6^{2}\right] \\
& =(1 / 144)[15240-36] \\
& =15204 / 144 \\
& =1267 / 12 \\
& =105.583
\end{aligned}
$$

Hence, standard deviation $=\sigma=\sqrt{ } 105.583$

$$
=10.275
$$

$\therefore$ Mean $=93$, variance $=105.583$ and Standard Deviation $=10.275$
10. The diameters of circles (in mm) drawn in a design are given below:

| Diameters | $33-36$ | $37-40$ | $41-44$ | $45-48$ | $49-52$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| No. of circles | 15 | 17 | 21 | 22 | 25 |

Calculate the standard deviation and mean diameter of the circles.
[Hint first make the data continuous by making the classes as 32.5-36.5, 36.5-40.5, 40.5-44.5, 44.5-48.5, 48.5-52.5 and then proceed.]

## Solution:-

Let the assumed mean, $A=42.5$ and $h=4$

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Let us make the table of the given data and append other columns after calculations.

| Height (class) | Number of <br> children <br> (Frequency <br> $\left.\mathbf{f}_{\mathbf{i}}\right)$ | Midpoint <br> $\mathbf{X}_{\mathbf{i}}$ | $\mathbf{Y}_{\mathbf{i}}=\left(\mathbf{x}_{\mathbf{i}}-\right.$ <br> $\mathbf{A}) / \mathbf{h}$ | $\mathbf{Y}_{\mathbf{i} \mathbf{2}}$ | $\mathbf{f}_{\mathbf{i} \mathbf{y}_{\mathbf{i}}}$ | $\mathbf{f}_{\mathbf{i} \mathbf{y}_{\mathrm{iz}}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $32.5-36.5$ | 15 | 34.5 | -2 | 4 | -30 | 60 |
| $36.5-40.5$ | 17 | 38.5 | -1 | 1 | -17 | 17 |
| $40.5-44.5$ | 21 | 42.5 | 0 | 0 | 0 | 0 |
| $44.5-48.5$ | 22 | 46.5 | 1 | 1 | 22 | 22 |
| $48.5-52.5$ | 25 | 50.5 | 2 | 4 | 50 | 100 |
|  | $\mathrm{~N}=100$ |  |  |  | 25 | 199 |

Mean,
$\overline{\mathrm{x}}=\mathrm{A}+\frac{\sum_{\mathrm{i}=1}^{\mathrm{a}} \mathrm{f}_{\mathrm{i}} \mathrm{y}_{\mathrm{i}}}{\mathrm{N}} \times \mathrm{h}$
Where, $\mathrm{A}=42.5, \mathrm{~h}=4$
So, $\bar{x}=42.5+(25 / 100) \times 4$

$$
\begin{aligned}
& =42.5+1 \\
& =43.5
\end{aligned}
$$

Then, Variance,

$$
\begin{aligned}
\sigma^{2} & =\frac{h^{2}}{N^{2}}\left[N \Sigma f_{\mathrm{i}} y_{\mathrm{i}}^{2}-\left(\Sigma \mathrm{f}_{\mathrm{i}} \mathrm{y}_{\mathrm{i}}\right)^{2}\right] \\
\sigma^{2} & =\left(4^{2} / 100^{2}\right)\left[100(199)-25^{2}\right] \\
& =(1 / 625)[19900-625] \\
& =19275 / 625 \\
& =771 / 25 \\
& =30.84
\end{aligned}
$$

Hence, standard deviation $=\sigma=\sqrt{ } 30.84$

$$
=5.553
$$

$\therefore$ Mean $=43.5$, variance $=30.84$ and Standard Deviation $=5.553$.

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## EXERCISE 15.3

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1. From the data given below state which group is more variable, $A$ or $B$ ?

| Marks | $10-20$ | $20-30$ | $30-40$ | $40-50$ | $50-60$ | $60-70$ | $70-80$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Group A | 9 | 17 | 32 | 33 | 40 | 10 | 9 |
| Group B | 10 | 20 | 30 | 25 | 43 | 15 | 7 |

## Solution:-

For comparing the variability or dispersion of two series, we calculate the coefficient of variance for each series. The series having greater C.V. is said to be more variable than the other. The series having lesser C.V. is said to be more consistent than the other.
Co-efficient of variation (C.V.) $=(\sigma / x) \times 100$
Where, $\sigma=$ standard deviation, $\bar{x}=$ mean
For Group A

| Marks | $\begin{aligned} & \text { Group } \\ & \text { A } \\ & \mathrm{f}_{\mathrm{i}} \\ & \hline \end{aligned}$ | Midpoint $\mathrm{X}_{\mathrm{i}}$ | $Y_{i}=\left(x_{i}-A\right) / h$ | $\left(Y_{i}\right)^{2}$ | fiy | $\mathrm{fi}_{\mathrm{i}}\left(\mathrm{y}_{\mathrm{i}}\right)^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} 10- \\ 20 \end{gathered}$ | 9 | 15 | $((15-45) / 10)=-3$ | $\begin{gathered} (- \\ 3)^{2} \\ =9 \\ \hline \end{gathered}$ | - | 81 |
| $\begin{gathered} 20- \\ 30 \end{gathered}$ | 17 | 25 | $((25-45) / 10)=-2$ | $\begin{gathered} (- \\ 2)^{2} \\ =4 \\ \hline \end{gathered}$ | - | 68 |

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Mean, $\bar{x}=A+\frac{\sum_{i=1}^{a} f_{i} y_{i}}{N} \times h$
Where $A=45$, and
$y_{i}=\left(x_{i}-A\right) / h$

Here $\mathrm{h}=$ class size $=20-10$

$$
h=10
$$

So, $x=45+((-6 / 150) \times 10)$

$$
\begin{aligned}
& =45-0.4 \\
& =44.6
\end{aligned}
$$

Then, variance $\sigma^{2}=\frac{h^{2}}{N^{2}}\left[N \Sigma f_{i} y_{i}^{2}-\left(\Sigma \mathrm{f}_{\mathrm{i}} \mathrm{y}_{\mathrm{i}}\right)^{2}\right]$

$$
\begin{aligned}
& \sigma^{2}=\left(10^{2} / 150^{2}\right)\left[150(342)-(-6)^{2}\right] \\
&=(100 / 22500)[51,300-36] \\
&=(100 / 22500) \times 51264 \\
&=227.84
\end{aligned}
$$

Hence, standard deviation $=\sigma=\sqrt{ } 227.84$

$$
=15.09
$$

$\therefore$ C.V for group $A=(\sigma / x) \times 100$

$$
\begin{aligned}
& =(15.09 / 44.6) \times 100 \\
& =33.83
\end{aligned}
$$

Now, for group B.

| Marks | Group B <br> $\mathbf{f}_{\mathbf{i}}$ | Mid-point <br> $\mathbf{X i}_{\mathbf{i}}$ | $\mathbf{Y}_{\mathbf{i}}=\left(\mathbf{x}_{\mathbf{i}}-\mathbf{A}\right) / \mathbf{h}$ | $\left.\mathbf{( Y}_{\mathbf{i}}\right)^{\mathbf{2}}$ | $\mathbf{f}_{\mathbf{i}} \mathbf{y}_{\mathbf{i}}$ | $\mathbf{f}_{\mathbf{i}}\left(\mathbf{y}_{\mathbf{i}}\right)_{\mathbf{2}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $10-20$ | 10 | 15 | $((15-45) / 10)$ <br> $=-3$ | $(-3)^{2}$ <br> $=9$ | -30 | 90 |
| $20-30$ | 20 | 25 | $((25-45) / 10)$ <br> $=-2$ | $(-2)^{2}$ <br> $=4$ | -40 | 80 |
| $30-40$ | 30 | 35 | $((35-45) / 10)$ <br> $=-1$ | $(-1)^{2}$ <br> $=1$ | -30 | 30 |
| $40-50$ | 25 | 45 | $((45-45) / 10)$ <br> $=0$ | $0^{2}$ | 0 | 0 |
| $50-60$ | 43 | 55 | $((55-45) / 10)$ <br> $=1$ | $1^{2}$ <br> $=1$ | 43 | 43 |
| $60-70$ | 15 | 65 | $((65-45) / 10)$ <br> $=2$ | $2^{2}$ <br> $=4$ | 30 | 160 |

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| $70-80$ | 7 | 75 | $((75-45) / 10)$ <br> $=3$ | $3^{2}$ <br> $=9$ | 21 | 189 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Total | 150 |  |  |  | -6 | 592 |

Mean, $\overline{\mathrm{x}}=\mathrm{A}+\frac{\sum_{\mathrm{i}=1}^{\mathrm{a}} \mathrm{f}_{\mathrm{i}} \mathrm{y}_{\mathrm{i}}}{\mathrm{N}} \times \mathrm{h}$
Where $A=45$,

$$
h=10
$$

So, $x=45+((-6 / 150) \times 10)$

$$
\begin{aligned}
& =45-0.4 \\
& =44.6
\end{aligned}
$$

Then, variance $\sigma^{2}=\frac{h^{2}}{N^{2}}\left[N \Sigma f_{i} y_{i}^{2}-\left(\Sigma \mathrm{f}_{\mathrm{i}} \mathrm{y}_{\mathrm{i}}\right)^{2}\right]$

$$
\begin{aligned}
\sigma^{2} & =\left(10^{2} / 150^{2}\right)\left[150(592)-(-6)^{2}\right] \\
& =(100 / 22500)[88,800-36] \\
& =(100 / 22500) \times 88,764 \\
& =394.50
\end{aligned}
$$

Hence, standard deviation $=\sigma=\sqrt{ } 394.50$

$$
=19.86
$$

$\therefore$ C.V for group $B=(\sigma / x) \times 100$

$$
\begin{aligned}
& =(19.86 / 44.6) \times 100 \\
& =44.53
\end{aligned}
$$

By comparing C.V. of group A and group B.
C.V of Group B > C.V. of Group A So,

Group $B$ is more variable.
2. From the prices of shares $X$ and $Y$ below, find out which is more stable in value:

| $X$ | 35 | 54 | 52 | 53 | 56 | 58 | 52 | 50 | 51 | 49 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $Y$ | 108 | 107 | 105 | 105 | 106 | 107 | 104 | 103 | 104 | 101 |

## Solution:-

From the given data,
Let us make the table of the given data and append other columns after calculations.

| $\mathbf{X}\left(\mathbf{x}_{\mathbf{i}}\right)$ | $\mathbf{Y}\left(\mathbf{y}_{\mathbf{i}}\right)$ | $\mathbf{X i 2}$ | $\mathbf{Y}_{\mathbf{i} 2}$ |
| :---: | :---: | :---: | :---: |


| 35 | 108 | 1225 | 11664 |
| :---: | :---: | :---: | :---: |
| 54 | 107 | 2916 | 11449 |
| 52 | 105 | 2704 | 11025 |
| 53 | 105 | 2809 | 11025 |
| 56 | 106 | 8136 | 11236 |
| 58 | 107 | 3364 | 11449 |
| 52 | 104 | 2704 | 10816 |
| 50 | 103 | 2500 | 10609 |
| 51 | 104 | 2601 | 10816 |
| 49 | 1050 | 2401 | 10201 |
| Total =510 | 26360 | 110290 |  |

We have to calculate Mean for $x$,
Mean $\bar{x}=\sum x_{i} / n$
Where, $\mathrm{n}=$ number of terms

$$
\begin{aligned}
& =510 / 10 \\
& =51
\end{aligned}
$$

$=51$
Then, Variance for x
$\frac{1}{\mathrm{n}^{2}}\left[\mathrm{~N} \sum \mathrm{x}_{\mathrm{i}}{ }^{2}-\left(\sum \mathrm{x}_{\mathrm{i}}\right)^{2}\right]$ =

$$
\begin{aligned}
& =\left(1 / 10^{2}\right)\left[(10 \times 26360)-510^{2}\right] \\
& =(1 / 100)(263600-260100) \\
& =3500 / 100 \\
& =35
\end{aligned}
$$

WKT Standard deviation $=$ Vvariance

$$
\begin{aligned}
& =\sqrt{ } 35 \\
& =5.91
\end{aligned}
$$

So, co-efficient of variation $=(\sigma / x) \times 100$

$$
\begin{aligned}
& =(5.91 / 51) \times 100 \\
& =11.58
\end{aligned}
$$

Now, we have to calculate Mean for y ,
Mean $\bar{y}=\Sigma y_{i} / n$

Where, $\mathrm{n}=$ number of terms

$$
\begin{aligned}
& =1050 / 10 \\
& =105
\end{aligned}
$$

$=105$
Then, Variance for y
$\frac{1}{\mathrm{n}^{2}}\left[\mathrm{~N} \sum \mathrm{y}_{\mathrm{i}}{ }^{2}-\left(\sum \mathrm{y}_{\mathrm{i}}\right)^{2}\right]$ =

$$
\begin{aligned}
& =\left(1 / 10^{2}\right)\left[(10 \times 110290)-1050^{2}\right] \\
& =(1 / 100)(1102900-1102500) \\
& =400 / 100 \\
& =4
\end{aligned}
$$

WKT Standard deviation $=$ Vvariance

$$
\begin{aligned}
& =\sqrt{ } 4 \\
& =2
\end{aligned}
$$

So, co-efficient of variation $=(\sigma / x) \times 100$

$$
\begin{aligned}
& =(2 / 105) \times 100 \\
& =1.904
\end{aligned}
$$

By comparing C.V. of X and Y .
C.V of $X>C . V$. of $Y$

So, Y is more stable than X .
3. An analysis of monthly wages paid to workers in two firms $A$ and $B$, belonging to the same industry, gives the following results:

|  | Firm A | Firm B |
| :---: | :---: | :---: |
| No. of wages earners | 586 | 648 |
| Mean of monthly wages | Rs 5253 | Rs 5253 |
| Variance of the distribution of wages | 100 | 121 |

(i) Which firm A or B pays larger amount as monthly wages?
(ii) Which firm, A or B, shows greater variability in individual wages? Solution:-

From the given table,
Mean monthly wages of firm A = Rs 5253 and
Number of wage earners $=586$
Then,
Total amount paid $=586 \times 5253$

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$$
\text { = Rs } 3078258
$$

Mean monthly wages of firm $B=$ Rs 5253
Number of wage earners $=648$
Then,
Total amount paid $=648 \times 5253$

$$
=\text { Rs 34,03,944 }
$$

(i) So, firm $B$ pays larger amount as monthly wages.
(ii) Variance of firm $A=100$

We know that, standard deviation $(\sigma)=\sqrt{ } 100$

$$
=10
$$

Variance of firm B=121
Then,
Standard deviation $(\sigma)=\mathrm{V}(121)$

$$
=11
$$

Hence the standard deviation is more in case of Firm B that means in firm B there is greater variability in individual wages.
4. The following is the record of goals scored by team $A$ in a football session:

| No. of goals <br> scored | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| No. of matches | 1 | 9 | 7 | 5 | 3 |

For the team B, mean number of goals scored per match was 2 with a standard deviation 1.25 goals. Find which team may be considered more consistent?

## Solution:-

From the given data,
Let us make the table of the given data and append other columns after calculations.

| Number of <br> goals scored $\mathbf{x}_{\mathbf{i}}$ | Number of <br> matches $\mathbf{f}_{\mathbf{i}}$ | $\mathbf{f i x i}_{\mathbf{i}}$ | $\mathbf{X}_{\mathbf{i 2}}$ | $\mathbf{f}_{\mathbf{i} \mathbf{X}_{\mathbf{i} 2}}$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 1 | 0 | 0 | 0 |
| 1 | 9 | 9 | 1 | 9 |
| 2 | 7 | 14 | 4 | 28 |
| 3 | 5 | 15 | 9 | 45 |

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| 4 | 3 | 12 | 16 | 48 |
| :---: | :---: | :---: | :---: | :---: |
| Total | 25 | 50 |  | 130 |

First we have to calculate Mean for Team A,
Mean $=\frac{\sum f_{i} x_{i}}{\sum f_{i}}=\frac{50}{25}=2$
Then,
Variance $=\frac{1}{N^{2}}\left[N \sum f_{i} x_{i}{ }^{2}-\left(\sum f_{i} x_{i}\right)^{2}\right]$

$$
=\frac{1}{25^{2}}[25 \times 130-2500]=\frac{750}{625}=1.2
$$

We know that, Standard deviation $\sigma=$ Vvariance $=\sqrt{ } 1.2=1.09$
Hence co-efficient of variation of team A,
C. $V \cdot{ }_{A}=\frac{\sigma}{\bar{x}} \times 100=\frac{1.09}{2} \times 100=54.5$

For team B
Given, $\bar{x}=2$
Standard deviation $\sigma=1.25$
So, co-efficient of variation of team B,
$\Rightarrow C . V \cdot B=\frac{1.25}{2} \times 100=62.5$
Since C.V. of firm $B$ is greater $:$.
Team A is more consistent.
5. The sum and sum of squares corresponding to length $x$ (in cm ) and weight $y$ (in gm) of 50 plant products are given below:

$$
\sum_{i=1}^{50} x_{i}=212, \quad \sum_{i=1}^{50} x_{i}^{2}=902.8, \quad \sum_{i=1}^{50} y_{i}=261, \sum_{i=1}^{50} y_{i}^{2}=1457.6
$$

Which is more varying, the length or weight?

## Solution:-

First we have to calculate Mean for Length x ,
Mean $=\overline{\mathrm{x}}=\frac{\sum \mathrm{x}_{\mathrm{i}}}{\mathrm{n}}=\frac{212}{50}=4.24$
Then,

$$
\begin{aligned}
\text { Variance } & =\frac{1}{\mathrm{~N}^{2}}\left[\mathrm{~N} \sum \mathrm{f}_{\mathrm{i}} \mathrm{x}_{\mathrm{i}}^{2}-\left(\sum \mathrm{f}_{\mathrm{i}} \mathrm{x}_{\mathrm{i}}\right)^{2}\right] \\
& =\left(1 / 50^{2}\right)\left[(50 \times 902.8)-212^{2}\right] \\
& =(1 / 2500)(45140-44944) \\
& =196 / 2500 \\
& =0.0784
\end{aligned}
$$

We know that, Standard deviation $\sigma=\sqrt{ }$ variance

$$
\begin{aligned}
& =\mathrm{V} 0.0784 \\
& =0.28
\end{aligned}
$$

Hence co-efficient of variation of team A,
C. $V_{\cdot x}=\frac{\sigma}{\bar{x}} \times 100=\frac{0.28}{4.24} \times 100=6.603$

Now we have to calculate mean of Weight y
$\bar{y}=\sum y_{i} / n$

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$$
\begin{aligned}
& =261 / 50 \\
& =5.22
\end{aligned}
$$

Then,

$$
\begin{aligned}
\text { Variance } & =\left(1 / N^{2}\right)\left[\left(N \Sigma f_{i} y_{i}^{2}\right)-\left(\sum f_{i} y_{i}\right)^{2}\right] \\
& =\left(1 / 50^{2}\right)\left[(50 \times 1457.6)-261^{2}\right] \\
& =(1 / 2500)(72880-68121) \\
& =4759 / 2500 \\
& =1.9036
\end{aligned}
$$

We know that, Standard deviation $\sigma=$ Vvariance

$$
\begin{aligned}
& =\mathrm{V} 1.9036 \\
& =1.37
\end{aligned}
$$

So, co-efficient of variation of team B,
C. $V_{\cdot} \cdot \frac{\sigma}{\bar{X}} \times 100=\frac{1.37}{5.22} \times 100=26.24$

Since C.V. of firm weight y is greater
$\therefore$ Weight is more varying.

## MISCELLANEOUS EXERCISE

1. The mean and variance of eight observations are 9 and 9.25 , respectively. If six of the observations are $6,7,10,12,12$ and 13 , find the remaining two observations. Solution:-
Form the question it is given that,
Variance of eight observations are 9 and 9.25 .
There are six observations given $6,7,10,12,12$, and 13
Let us assume the remaining two observations to be x and y respectively such that,

Observations: 6, 7, 10, 12, 12, 13, x, y.
We have to calculate the mean of given observations,
$\therefore$ Mean, $\overline{\mathrm{x}}=\frac{6+7+10+12+12+13+\mathrm{x}+\mathrm{y}}{8}$
$9=\frac{6+7+10 \stackrel{8}{+} 12+12+13+x+y}{8}$

$$
60+x+y=72
$$

$$
x+y=12
$$

Now, Variance $=\frac{1}{\mathrm{n}} \sum_{\mathrm{i}=1}^{8}\left(\mathrm{x}_{\mathrm{i}}-\mathrm{x}\right)^{2}$
$9.25=\frac{1}{8}\left[(-3)^{2}+(-2)^{2}+1^{2}+3^{2}+4^{2}+x^{2}+y^{2}-18(x+y)+2 \times 9^{2}\right]$
By using equation (j) substitute 12 instead of ( $\mathrm{x}+\mathrm{y}$ )
$9.25=\frac{1}{8}\left[9+4+1+9+9+16+x^{2}+y^{2}-18 \times 12+162\right]$
$9.25=\frac{1}{8}\left[48+\mathrm{x}^{2}+\mathrm{y}^{2}-216+162\right]$
$9.25=\frac{1}{8}\left[\mathrm{x}^{2}+\mathrm{y}^{2}-6\right]$
$x^{2}+y^{2}=80$
... [we call it as equation (ii)]

So, from equation (i) we have:

$$
x^{2}+y^{2}+2 x y=144
$$

Thus, from (ii) and (iii), we have

$$
2 x y=64 \text { (iv) }
$$

Now by subtracting (iv) from (ii), we get:

$$
\begin{aligned}
& x^{2}+y^{2}-2 x y=80-64 \\
& x-y= \pm 4(v)
\end{aligned}
$$

Hence, from equation (i) and (v) we have:
When $x-y=4$
Then, $x=8$ and $\mathrm{y}=4$
And, when $x-y=-4$
Then, $x=4$ and $y=8$
$\therefore$ The remaining observations are 4 and 8
2. The mean and variance of 7 observations are 8 and 16 , respectively. If five of the observations are $\mathbf{2 , 4 , 1 0 , 1 2 , 1 4}$. Find the remaining two observations. Solution:Form the question it is given that,

Variance of seven observations are 8 and 16.
There are six observations given $2,4,10,12$, and 14
Let us assume the remaining two observations to be x and y respectively such that,

Observations: $2,4,10,12,14, x, y$.
We have to calculate the mean of given observations,
$\therefore$ Mean, $\overline{\mathrm{x}}=\frac{2+4+10+12+14+\mathrm{x}+\mathrm{y}}{7}=8$

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$$
x+y=14 \quad \ldots[\text { we call it as equation (i) }]
$$

In the question it is given that,
Variance $=16$
We know that,
Variance $=\frac{1}{\mathrm{n}} \sum_{\mathrm{i}=1}^{\mathrm{n}}\left(\mathrm{x}_{1}-\overline{\mathrm{x}}\right)^{2}$

$$
\begin{aligned}
16=\frac{1}{7}\left[(-6)^{2}\right. & +(-4)^{2}+(2)^{2}+(4)^{2}+(6)^{2}+x^{2}+y^{2}-2 \times 8(x+y) \\
& \left.+2 \times(8)^{2}\right]
\end{aligned}
$$

By using equation (i) substitute 14 instead of ( $x+y$ )

$$
\begin{aligned}
& 16=\frac{1}{7}\left[36+16+4+16+36+x^{2}+y^{2}-16(14)+2(64)\right] \\
& 16=\frac{1}{7}\left[12+x^{2}+y^{2}\right] \\
& x^{2}+y^{2}=112-12 \\
& x^{2}+y^{2}=100 \quad \ldots . \text { we call it as equation (ii)] }
\end{aligned}
$$

So, from equation (i) we have:

$$
x^{2}+y^{2}+2 x y=196 \quad \ldots[\text { we call it as equation (iii) }]
$$

Thus, from equation (ii) and (iii) we have:

$$
\begin{aligned}
& 2 x y=196-100 \\
& 2 x y=96 \text { (iv) }
\end{aligned}
$$

Now subtracting equation (iv) from (ii),
We get:

$$
\begin{aligned}
& x^{2}+y^{2}-2 x y=100-96 \\
& (x-y)^{2}=4 \\
& x-y= \pm 2(v)
\end{aligned}
$$

Hence, from equation (i) and (v) we have:
When $x-y=2$ then $x=8$ and $y=6$
And, when $x-y=-2$ then $x=6$ and $y=8$ : the remaining observations are 6 and 8
3. The mean and standard deviation of six observations are 8 and 4, respectively. If each observation is multiplied by 3 , find the new mean and new standard deviation of the resulting observations.

## Solution:-

From the question it is given that,
Mean of six observations $=8$
Standard deviation of six observations $=4$
Let us assume the observations be $\mathrm{x}_{1}, \mathrm{x}_{2}, \mathrm{x}_{3}, \mathrm{x}_{4}, \mathrm{x}_{5}$ and $\mathrm{x}_{6}$
So, mean of assumed observations,

$$
\therefore \text { Mean } \bar{x}=\frac{x_{1}+x_{2}+x_{3}+x_{4}+x_{5}+x_{6}}{6}=8
$$

Then, as per the question if each observation is multiplied by 3 and the resulting observations are $y_{i}$ then, we have:

$$
y_{i}=3 x_{i}
$$

Hence, $x_{i}=\frac{1}{3} y_{i}($ For $i=1$ to 6$)$
$\therefore$ New mean, $\overline{\mathrm{y}}=\frac{\mathrm{y}_{1}+\mathrm{y}_{2}+\mathrm{y}_{3}+\mathrm{y}_{4}+\mathrm{y}_{5}+\mathrm{y}_{6}}{6}$

$$
\begin{aligned}
& =\frac{3\left(x_{1}+x_{2}+x_{3}+x_{4}+x_{5}+x_{6}\right)}{6} \\
& =3 \times 8 \\
& =24
\end{aligned}
$$

We know that,
Standard deviation $(\sigma)=\sqrt{\frac{1}{n} \sum_{i=1}^{6}\left(x_{i}-\bar{x}\right)^{2}}$
By squaring on both the sides

$$
\therefore(4)^{2}=\frac{1}{6} \sum_{\mathrm{i}=1}^{6}\left(\mathrm{x}_{\mathrm{i}}-\overline{\mathrm{x}}\right)^{2}
$$

$$
\begin{equation*}
\sum_{i=1}^{6}\left(x_{i}-\bar{x}\right)^{2}=96 \tag{ii}
\end{equation*}
$$

Hence, from (i) and (ii) we have:

$$
\begin{aligned}
& \bar{y}=3 \bar{x} \\
& \bar{x}=\frac{1}{3} \bar{y}
\end{aligned}
$$

Now, by substituting the values of $\mathrm{x}_{\mathrm{i}}$ and $\overline{\mathrm{x}}$ in (ii) we have:

$$
\sum_{i=1}^{6}\left(\frac{1}{3} y_{i}-\frac{1}{3} \bar{y}\right)^{2}=96
$$

Thus, $\sum_{i=1}^{6}\left(y_{i}-\bar{y}\right)^{2}=864$
So, the variance of new observation $=(1 / 6) \times 864$

$$
=144
$$

Therefore, standard deviation of new observation $=\sqrt{144}$

$$
=12
$$

4. Given that $\bar{x}$ is the mean and $\sigma^{2}$ is the variance of $n$ observations $x_{1}, x_{2}, \ldots, x_{n}$. Prove that the mean and variance of the observations $\mathrm{ax}_{1}, \mathrm{ax}_{2}, \mathrm{ax}_{3}, \ldots ., \mathrm{ax}$ are $\mathrm{a} \overline{\mathrm{x}}$ and $a^{2} \sigma^{2}$, respectively, $(a \neq 0)$.
Solution:-
From the question it is given that, n observations are $\mathrm{x}_{1}, \mathrm{x}_{2}, \ldots . . \mathrm{x}_{\mathrm{n}}$
Mean of the $n$ observation $=\bar{x}$
Variance of the $n$ observation $=\sigma^{2}$
As we know that,

Variance, $\quad \sigma^{2}=\frac{1}{\mathrm{n}} \sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{y}_{1}\left(\mathrm{x}_{\mathrm{i}}-\overline{\mathrm{x}}\right)^{2} \quad \ldots[$ equation (i)]
As per the condition given in the question, if each of the observation is being multiplied by ' $a$ ' and the new observation are $y_{i}$ the, we have:

$$
y_{i}=a x_{i}
$$

Thus, $\mathrm{x}_{\mathrm{i}}=\frac{1}{\mathrm{a}} \mathrm{y}_{\mathrm{i}}$

$$
\begin{aligned}
& \therefore \overline{\mathrm{y}}=\frac{1}{\mathrm{n}} \sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{y}_{\mathrm{i}} \\
& \overline{\mathrm{y}}=\frac{1}{\mathrm{n}} \sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{ax}_{\mathrm{i}} \\
& \overline{\mathrm{y}}=\frac{\mathrm{a}}{\mathrm{n}} \sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{x}_{\mathrm{i}} \\
& \overline{\mathrm{y}}=\mathrm{a} \overline{\mathrm{x}}
\end{aligned}
$$

Therefore, mean of the observations $\mathrm{ax}_{1}, \mathrm{ax}_{2}, \ldots . . \mathrm{ax}_{n}$ is a $\overline{\mathrm{x}}$
Now, by substituting the values of $x_{i}$ and $\bar{x}$ in equation(i), we get:

$$
\begin{aligned}
& \sigma^{2}=\frac{1}{n} \sum_{i=1}^{n}\left(\frac{1}{a} y_{1}-\frac{1}{a} \bar{y}\right)^{2} \\
& a^{2} \sigma^{2}=\frac{1}{n} \sum_{i=1}^{n}\left(y_{1}-\bar{y}\right)^{2}
\end{aligned}
$$

$\therefore$ the variance of the given observations $\mathrm{ax}_{1}, \mathrm{ax}_{2}, \ldots . \mathrm{ax}_{n}$ is $\mathrm{a}^{2} \sigma^{2}$
5. The mean and standard deviation of 20 observations are found to be 10 and 2 , respectively. On rechecking, it was found that an observation 8 was incorrect.
Calculate the correct mean and standard deviation in each of the following cases: (i) If wrong item is omitted. (ii) If it is replaced by 12

## Solution:-

(i) If wrong item is omitted,

From the question it is given that,
The number of observations i.e. $\mathrm{n}=20$
The incorrect mean $=20$
The incorrect standard deviation $=2$

$$
\begin{aligned}
& \overline{\mathrm{X}}=\frac{1}{\mathrm{n}} \sum_{\mathrm{i}=1}^{20} \mathrm{X}_{1} \\
& 10=\frac{1}{20} \sum_{\mathrm{i}=1}^{20} \mathrm{X}_{1} \\
& \sum_{\mathrm{i}=1}^{20} \mathrm{X}_{1}=200
\end{aligned}
$$

By the calculation the incorrect sum of observations $=200$
Hence, correct sum of observations $=200-8$

$$
=192
$$

Therefore the correct mean $=$ correct sum/19

$$
\begin{aligned}
& =192 / 19 \\
& =10.1
\end{aligned}
$$

We know that, Standard deviation $(\sigma)=\sqrt{\frac{1}{n} \sum_{i=1}^{n} X_{1}-\frac{1}{n^{2}}\left(\sum_{i=1}^{n} X_{1}\right)^{2}}$

$$
2=\sqrt{\frac{1}{n} \sum_{i=1}^{n} X_{1}^{2}-(\overline{\mathrm{X}})^{2}}
$$

$4=\frac{1}{20}$ Incorrect $\sum_{i=1}^{\mathrm{n}} \mathrm{X}_{1}^{2}-100$
Incorrect $\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{X}_{1}^{2}=2080$
Therefore, correct $\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{X}_{1}^{2}=$ Incorrect $\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{X}_{1}^{2}-(8)^{2}$

$$
\begin{aligned}
& =2080-64 \\
& =2016
\end{aligned}
$$

Finally we came to calculate correct standard deviation,

$$
\begin{aligned}
\text { Hence, Correct standard deviation } & =\sqrt{\frac{\text { Correct } \sum \mathrm{X}_{1}^{2}}{\mathrm{n}}-(\text { Correct Mean })^{2}} \\
& =\sqrt{\frac{2016}{19}-(10.1)^{2}} \\
& =\sqrt{1061.1-102.1} \\
& =2.02
\end{aligned}
$$

(ii) If it is replaced by 12,

From the question it is given that,
The number of incorrect sum observations i.e. $\mathrm{n}=200$
The correct sum of observations $n=200-8+12 n=$ 204
Then, correct mean $=$ correct sum/20

$$
\begin{aligned}
& =204 / 20 \\
& =10.2
\end{aligned}
$$

Standard deviation $(\sigma)=\sqrt{\frac{1}{\mathrm{n}} \sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{X}_{1}-\frac{1}{\mathrm{n}^{2}}\left(\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{X}_{1}\right)^{2}}$

$$
\therefore 2=\sqrt{\frac{1}{\mathrm{n}} \sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{X}_{1}^{2}-(\overline{\mathrm{X}})^{2}}
$$

$4=\frac{1}{20}$ Incorrect $\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{X}_{1}^{2}-100$
Incorrect $\sum_{i=1}^{n} X_{1}^{2}=2080$
Thus, correct $\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{X}_{1}^{2}=\operatorname{Incorrect} \sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{X}_{1}^{2}-(8)^{2}+(12)^{2}$

$$
\begin{aligned}
& =2080-64+144 \\
& =2160
\end{aligned}
$$

Hence, Correct standard deviation $=\sqrt{\frac{\text { Correct } \sum \mathrm{X}_{1}^{2}}{\mathrm{n}}-(\text { Correct Mean })^{2}}$

$$
\begin{aligned}
& =\sqrt{\frac{2160}{20}-(10.2)^{2}} \\
& =\sqrt{108-104.04} \\
& =\sqrt{3.96} \\
& =1.98
\end{aligned}
$$

6. The mean and standard deviation of marks obtained by 50 students of a class in three subjects, Mathematics, Physics and Chemistry are given below:

| Subject | Mathematics | Physics | Chemistry |
| :---: | :---: | :---: | :---: |
| Mean | 42 | 32 | 40.9 |
| Standard deviation | 12 | 15 | 20 |

Which of the three subjects shows the highest variability in marks and which shows the lowest?

## Solution:-

From the question it is given that,
Mean of Mathematics $=42$
Standard deviation of Mathematics $=12$
Mean of Physics = 32
Standard deviation of physics $=15$
Mean of Chemistry $=40.9$
Standard deviation of chemistry $=20$
As we know that,
Coefficient of variation (C.V) $=\frac{\text { Standard deviation }}{\text { Mean }} \times 100$
Then,
C.V. in Mathematics $=(12 / 42) \times 100$

$$
=28.57
$$

C.V. in Mathematics $=(15 / 32) \times 100$

$$
=46.87
$$

C.V. in Mathematics $=(20 / 40.9) \times 100$

$$
=48.89
$$

Hence, subject with highest variability in marks is chemistry as subject with the greater C.V is more variable than others
7. The mean and standard deviation of a group of 100 observations were found to be 20 and 3 , respectively. Later on it was found that three observations were incorrect, which were recorded as 21,21 and 18 . Find the mean and standard deviation if the incorrect observations are omitted.

## Solution:-

From the question it is given that,
The total number of observations $(\mathrm{n})=100$
Incorrect mean, ( x ) $=20$
And, Incorrect standard deviation $(\sigma)=3$

$$
\therefore 20=\frac{1}{100} \sum_{\mathrm{i}=1}^{100} \mathrm{X}_{1}
$$

By cross multiplication, we get

$$
\begin{aligned}
& \sum_{i=1}^{100} x_{1}=20 \times 100 \\
& \sum_{i=1}^{100} x_{1}=2000
\end{aligned}
$$

Hence, incorrect sum of observations is 2000
Now, correct sum of observations $=2000-21-21-18$

$$
\begin{aligned}
& =2000-60 \\
& =1940
\end{aligned}
$$

Therefore correct Mean $=$ Correct sum $/(100-3)$

$$
\begin{aligned}
& =1940 / 97 \\
& =20
\end{aligned}
$$

$\begin{aligned} &=20 \\ & \text { We know that, Standard deviation }(\sigma)=\sqrt{\frac{1}{n} \sum_{i=1}^{n} X_{1}-\frac{1}{n^{2}}\left(\sum_{i=1}^{n} \mathrm{X}\right)^{2}} \\ & 3=\sqrt{\frac{1}{n} \sum_{i=1}^{n} X_{1}^{2}-(\overline{\mathrm{X}})^{2}}\end{aligned}$

$$
3=\sqrt{\frac{1}{100} \times \text { Incorrect } \sum X_{1}^{2}-(20)^{2}}
$$

$$
\begin{aligned}
& \text { Incorrect } \sum \mathrm{X}_{1}^{2}=100(9+400) \\
& \text { Incorrect } \sum_{\text {Correct } \mathrm{X}_{1}^{2}}^{\sum_{i=1}^{\mathrm{n}} \mathrm{X}_{1}^{2}}=40900 \\
& \\
& =40900-441-441-324 \\
& \\
& =40900-1206 \\
& \\
& =39694
\end{aligned}
$$

Hence, correct standard deviation $=\sqrt{\frac{\text { Correct } \sum \mathrm{X}_{1}^{2}}{\mathrm{n}}-(\text { Correct mean })^{2}}$

$$
\begin{aligned}
& =\sqrt{\frac{39694}{97}-(20)^{2}} \\
& =\sqrt{409.216-400} \\
& =3.036
\end{aligned}
$$

