Exercise 5.3 Page: 112

1. Find the sum of the following APs.
(i) 2, 7, 12 ,…., to 10 terms.
(ii) − 37, − 33, − 29 ,…, to 12 terms
(iii) 0.6, 1.7, 2.8 ,…….., to 100 terms
(iv) 1/15, 1/12, 1/10, ... , to 11 terms

Solutions:

(i) Given, 2, 7, 12 , to 10 terms
For this A.P.,
first term, $a = 2$
And common difference, $d = a_2 - a_1 = 7 - 2 = 5$ $n = 10$

We know that, the formula for sum of nth term in AP series is,
$S_n = n/2\ [2a + (n - 1)\ d]$
$S_{10} = 10/2\ [2(2) + (10 - 1) \times 5]$
$= 5[4 + (9) \times (5)]$
$= 5 \times 49 = 245$

(ii) Given, −37, −33, −29 , to 12 terms For this A.P.,
first term, $a = -37$
And common difference, $d = a_2 - a_1 = (-33) - (-37)$
$= -33 + 37 = 4$ n
$= 12$

We know that, the formula for sum of nth term in AP series is,
$S_n = n/2\ [2a + (n - 1)\ d]$
$S_{12} = 12/2\ [2(-37) + (12 - 1) \times 4]$
$= 6[-74 + 11 \times 4]$
$= 6[-74 + 44]$
$= 6(-30) = -180$

(iii) Given, 0.6, 1.7, 2.8 , to 100 terms
For this A.P.,
first term, $a = 0.6$
and
Common difference, $d = a_2 - a_1 = 1.7 - 0.6 = 1.1$ n
$= 100$

We know that, the formula for sum of nth term in
AP series is, $S_n = n/2\ [2a + (n - 1)\ d]$
$S_{12} = 50/2\ [1.2 + (99) \times 1.1]$
$= 50[1.2 + 108.9]$
$= 50[110.1]$
$= 5505$

(iv) Given, 1/15, 1/12, 1/10, to 11 terms
For this A.P.,
First term, a = 1/5
Common difference, $d = a_2 - a_1 = \frac{1}{12} - \frac{1}{15} = \frac{1}{60}$

And number of terms = 11

We know that, the formula for sum of nth term in AP series is,
$S_n = n/2\ [2a + (n - 1)\ d]$

$$Sn = \frac{11}{2}\ [2\left(\frac{1}{15}\right) + \frac{(11-1)\,1}{60}]$$
$$= \frac{11}{2}[\frac{2}{15} + \frac{10}{60}]$$
$$= \frac{11}{2}[\frac{9}{30}]$$
$$= \frac{33}{20}$$

2. Find the sums given below:
(i) 7 +10$\frac{1}{2}$ + 14 + ... +84
(ii) 34 + 32 + 30 + ……….. + 10
(iii) − 5 + (− 8) + (− 11) + ………… + (− 230)

Solutions:

(i) For this given A.P., $7 + 10\frac{1}{2} + 14 + \ldots\ldots + 84$,
First term, a = 7 *nth term*, $a_n = 84$
Common difference, d $= a_2 - a_1 = \mathbf{10\frac{1}{2}} - 7 = \frac{21}{2} - 7 = \frac{7}{2}$
Let 84 be the n^{th} term of this A.P., then as per the nth term formula,
$a_n = a\ (n - 1)d$ $84 = 7 + (n - 1) \times 7/2$
$77 = (n - 1) \times 7/2$
$22 = n - 1$ n
$= 23$

We know that, sum of nth term is;
$S_n = n/2\ (a + l)$
$S_n = 23/2\ (7 + 84)$
$S_n = (23 \times 91/2) = 2093/2$
$S_n = 1046\frac{1}{2}$

(ii) Given, 34 + 32 + 30 + ……….. + 10
For this A.P., *first term*, $a = 34$
common difference, $d = a_2 - a_1 = 32 - 34 = -2$
nth term, $a_n = 10$
Let 10 be the n^{th} term of this A.P., therefore,
$a_n = a + (n - 1)\ d$ $10 = 34 + (n - 1)\ (-2)$
$-24 = (n - 1)\ (-2)$ $12 = n - 1$ $n = 13$

We know that, sum of nth term is;
$S_n = n/2\ (a + l)$
$= 13/2\ (34 + 10)$
$= (13 \times 44/2) = 13 \times 22$
$= 286$

(iii) Given, (−5) + (−8) + (−11) + ………… + (−230)
For this A.P., *First*
term, $a = -5$ *nth*
term, $a_n = -230$
Common difference, $d = a_2 - a_1 = (-8) - (-5)$
=> $d = -8 + 5 = -3$

Let −230 be the n^{th} term of this A.P., and by the nth term formula we know,

$a_n = a + (n - 1)d$
$-230 = -5 + (n - 1)\ (-3)$
$-225 = (n - 1)\ (-3)$
$(n - 1) = 75$ n
$= 76$

And, Sum of nth term,
$S_n = n/2\ (a + l)$
$= 76/2\ [(-5) + (-230)]$
$= 38(-235)$
$= -8930$

3. In an AP

(i) Given a = 5, d = 3, a_n = 50, find n and S_n.
(ii) Given a = 7, a_{13} = 35, find d and S_{13}.
(iii) Given a_{12} = 37, d = 3, find a and S_{12}.
(iv) Given a_3 = 15, S_{10} = 125, find d and a_{10}.
(v) Given d = 5, S_9 = 75, find a and a_9.
(vi) Given a = 2, d = 8, S_n = 90, find n and a_n.
(vii) Given a = 8, a_n = 62, S_n = 210, find n and d.
(viii) Given a_n = 4, d = 2, S_n = − 14, find n and a.
(ix) Given a = 3, n = 8, S = 192, find d.
(x) Given l = 28, S = 144 and there are total 9 terms. Find a.

Solutions:

(i) Given that, $a = 5$, $d = 3$, $a_n = 50$
As we know, from the formula of the nth term in an AP, $a_n = a + (n - 1)d$,
Therefore, putting the given values, we get,
$\Rightarrow 50 = 5 + (n - 1) \times 3$
$\Rightarrow 3(n - 1) = 45$
$\Rightarrow n - 1 = 15$
$\Rightarrow n = 16$

Now, sum of nth term,
$S_n = n/2\ (a + a_n)$
$S_n = 16/2\ (5 + 50) = 440$

(ii) Given that, $a = 7$, $a_{13} = 35$
As we know, from the formula of the nth term in an AP, $a_n = a + (n - 1)d$,
Therefore, putting the given values, we get,
$\Rightarrow 35 = 7 + (13 - 1)d$
$\Rightarrow 12d = 28$
$\Rightarrow d = 28/12 = 2.33$
Now, $S_n = n/2\ (a + a_n)$
$S_{13} = 13/2\ (7 + 35) = 273$

(iii)Given that, $a_{12} = 37$, $d = 3$
As we know, from the formula of the nth term in an AP,
$a_n = a + (n - 1)d$,

Therefore, putting the given values, we get,
$\Rightarrow a_{12} = a + (12 - 1)3$
$\Rightarrow 37 = a + 33$
$\Rightarrow a = 4$
Now, sum of nth term,
$S_n = n/2\ (a + a_n)$
$S_n = 12/2\ (4 + 37)$
$= 246$

(iv) Given that, $a_3 = 15$, $S_{10} = 125$
As we know, from the formula of the nth term in an AP,
$a_n = a + (n - 1)d$,
Therefore, putting the given values, we get, a_3
$= a + (3 - 1)d$
$15 = a + 2d$ ………………………….. **(i)**

Sum of the nth term,
$S_n = n/2\ [2a + (n - 1)d]$
$S_{10} = 10/2\ [2a + (10 - 1)d]$
$125 = 5(2a + 9d)$
$25 = 2a + 9d$ ……………………….. **(ii)**

On multiplying equation **(i)** by **(ii)**, we will get;
$30 = 2a + 4d$ .... **(iii)**

By subtracting equation **(iii)** from **(ii)**, we get,
$-5 = 5d$
$d = -1$

From equation **(i)**,
$15 = a + 2(-1)$ 15
$= a - 2$ $a = 17 =$
First term $a_{10} = a$
$+ (10 - 1)d$ $a_{10} =$
$17 + (9)(-1)$
$a_{10} = 17 - 9 = 8$

(v) Given that, $d = 5$, $S_9 = 75$
As, sum of nth terms in AP is,

$S_n = n/2\ [2a + (n - 1)d]$
Therfore, the sum of first nine terms are;
$S_9 = 9/2\ [2a + (9 - 1)5]$

$25 = 3(a + 20)$
$25 = 3a + 60$
$3a = 25 - 60$
$a = -35/3$

As we know, the nth term can be written as;
$a_n = a + (n - 1)d$ $a_9 = a + (9 - 1)(5)$
$= -35/3 + 8(5)$
$= -35/3 + 40$
$= (35+120/3) = 85/3$

(vi) Given that, $a = 2$, $d = 8$, $S_n = 90$

As, sum of nth term in an AP is,
$S_n = n/2\,[2a + (n - 1)d]$
$90 = n/2\,[2a + (n - 1)d]$
$\Rightarrow 180 = n(4 + 8n - 8) = n(8n - 4) = 8n^2 - 4n$
$\Rightarrow 8n^2 - 4n - 180 = 0$
$\Rightarrow 2n^2 - n - 45 = 0$
$\Rightarrow 2n^2 - 10n + 9n - 45 = 0$
$\Rightarrow 2n(n - 5) + 9(n - 5) = 0$
$\Rightarrow (2n - 9)(2n + 9) = 0$
So, $n = 5$ (as it is positive integer)
$\therefore a_5 = 8 + 5 \times 4 = 34$

(vii) Given that, $a = 8$, $a_n = 62$, $S_n = 210$
As, sum of nth term in an AP is,
$S_n = n/2\,(a + a_n)$
$210 = n/2\,(8 + 62)$
$\Rightarrow 35n = 210$
$\Rightarrow n = 210/35 = 6$

Now, $62 = 8 + 5d$
$\Rightarrow 5d = 62 - 8 = 54$
$\Rightarrow d = 54/5 = 10.8$

(viii) Given that, nth term, $a_n = 4$, common difference, $d = 2$, sum of nth term, $S_n = -14$.
As we know, from the formula of the nth term in an AP,

$a_n = a + (n - 1)d$,
Therefore, putting the given values, we get,

$4 = a + (n - 1)2$
$4 = a + 2n - 2$ a
$+ 2n = 6$
$a = 6 - 2n$ .... **(i)**

As we know, the sum of nth term is;
$S_n = n/2\ (a + a_n)$
$-14 = n/2\ (a + 4)$
$-28 = n\ (a + 4)$
$-28 = n\ (6 - 2n + 4)$ {From equation **(i)**}
$-28 = n\ (-2n + 10)$
$-28 = -2n^2 + 10n$ $2n^2$
$- 10n - 28 = 0$ $n^2 - 5n$
$-14 = 0$ $n^2 - 7n + 2n -$
$14 = 0$ $n\ (n - 7) + 2(n$
$- 7) = 0$
$(n - 7)\ (n + 2) = 0$

Either $n - 7 = 0$ or $n + 2 = 0$ n
$= 7$ or $n = -2$
However, n can neither be negative nor fractional.
Therefore, $n = 7$ From
equation **(i)**, we get $a = 6$
$- 2n$ $a = 6 - 2(7)$
$= 6 - 14$
$= -8$

(ix) Given that, first term, $a = 3$,
Number of terms, $n = 8$
And sum of nth term, $S = 192$

As we know,
$S_n = n/2\ [2a + (n - 1)d]$
$192 = 8/2\ [2 \times 3 + (8 - 1)d]$
$192 = 4\ [6 + 7d]$
$48 = 6 + 7d$
$42 = 7d$
$d = 6$

(x) Given that, $l = 28$, $S = 144$ and there are total of 9 terms.
Sum of nth term formula,
$S_n = n/2\ (a + l)$
$144 = 9/2\ (a + 28)$

$(16) \times (2) = a + 28$ 32
$= a + 28$
$a = 4$

4. How many terms of the AP. 9, 17, 25 … must be taken to give a sum of 636?

Solutions:

Let there be n terms of the AP. 9, 17, 25 …
For this A.P.,
First term, $a = 9$
Common difference, $d = a_2 - a_1 = 17 - 9 = 8$
As, the sum of nth terms, is;
$S_n = n/2\ [2a + (n - 1)d]$
$636 = n/2\ [2 \times a + (8 - 1) \times 8]$
$636 = n/2\ [18 + (n- 1) \times 8]$
$636 = n\ [9 + 4n - 4]$ 636
$= n\ (4n + 5)$
$4n^2 + 5n - 636 = 0$ $4n^2 + 53n$
$- 48n - 636 = 0$ $n\ (4n + 53) -$
$12\ (4n + 53) = 0$
$(4n + 53)\ (n - 12) = 0$

Either $4n + 53 = 0$ or $n - 12 = 0$ $n = (-53/4)$ or $n = 12$ n
cannot be negative or fraction, therefore, $n = 12$ only.

5. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Solution: Given that,
first term, $a = 5$ *last*
term, $l = 45$ *Sum of the*
AP, $S_n = 400$
As we know, the sum of AP formula is;
$S_n = n/2\ (a + l)$
$400 = n/2\ (5 + 45)$
$400 = n/2\ (50)$
Number of terms, $n = 16$
As we know, the last term of AP series can be written as;
$l = a + (n - 1)\ d$ $45 = 5 + (16 - 1)\ d$
$40 = 15d$
Common difference, $d = 40/15 = 8/3$

6. The first and the last term of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Solution: Given that,
First term, $a = 17$
Last term, $l = 350$
Common difference, $d = 9$

Let there be n terms in the A.P., thus the formula for last term can be written as;
$l = a + (n - 1)\, d$
$350 = 17 + (n - 1)9$
$333 = (n - 1)9$
$(n - 1) = 37\ n$
$= 38\ S_n = n/2$
$(a + l)$
$S_{38} = 13/2\ (17 + 350)$
$= 19 \times 367$
$= 6973$
Thus, this A.P. contains 38 terms and the sum of the terms of this A.P. is 6973.

7. Find the sum of first 22 terms of an AP in which $d = 7$ and 22nd term is 149.

Solution:Given,
Common difference, $d = 7$
22nd term, $a_{22} = 149$
Sum of first 22 term, $S_{22} = ?$
By the formula of nth term,
$a_n = a + (n - 1)d\ a_{22} = a +$
$(22 - 1)d\ 149 = a + 21 \times 7$
$149 = a + 147$
$a = 2 =$ First term

Sum of nth term,
$S_n = n/2\ (a + a_n)$
$= 22/2\ (2 + 149)$
$= 11 \times 151$
$= 1661$

8. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Solution: Given that,
Second term, $a_2 = 14$
Third term, $a_3 = 18$
Common difference, $d = a_3 - a_2 = 18 - 14 = 4$
$a_2 = a + d$
$14 = a + 4$
$a = 10$ = First term

Sum of nth term;
$S_n = n/2\ [2a + (n - 1)d]$
$S_{51} = 51/2\ [2 \times 10 + (51 - 1) \times 4]$
$= 51/2\ [2 + (20) \times 4]$
$= 51 \times 220/2$
$= 51 \times 110$
$= 5610$

9. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first *n* terms.

Solution: Given that,
$S_7 = 49$ S_{17}
$= 289$
We know, Sum of nth term;
$S_n = n/2\ [2a + (n - 1)d]$

Therefore,
$S_7 = 7/2\ [2a + (n - 1)d]$
$S_7 = 7/2\ [2a + (7 - 1)d]$
$49 = 7/2\ [2a + 16d]$
$7 = (a + 3d)$
$a + 3d = 7$ .... **(i)**

In the same way,
$S_{17} = 17/2\ [2a + (17 - 1)d]$
$289 = 17/2\ (2a + 16d)$
$17 = (a + 8d)$
$a + 8d = 17$ .... **(ii)**

Subtracting equation **(i)** from equation **(ii)**,
$5d = 10$ d
$= 2$
From equation **(i)**, we can write it as;
$a + 3(2) = 7$ $a + 6 = 7$

$a = 1$

Hence,
$S_n = n/2\ [2a + (n - 1)d]$
$= n/2\ [2(1) + (n - 1) \times 2]$
$= n/2\ (2 + 2n - 2)$
$= n/2\ (2n)$
$= n^2$

10. Show that $a_1, a_2 \ldots, a_n, \ldots$ form an AP where a_n is defined as below
(i) $a_n = 3 + 4n$ (ii) $a_n = 9 - 5n$
Also find the sum of the first 15 terms in each case.

Solutions:

(i) $a_n = 3 + 4n$ $a_1 = 3 +$
$4(1) = 7$ $a_2 = 3 + 4(2) = 3$
$+ 8 = 11$ $a_3 = 3 + 4(3) = 3$
$+ 12 = 15$ $a_4 = 3 + 4(4) = 3$
$+ 16 = 19$

We can see here, the common difference between the terms are;

$a_2 - a_1 = 11 - 7 = 4$ a_3
$- a_2 = 15 - 11 = 4$ a_4
$- a_3 = 19 - 15 = 4$

Hence, $a_{k+1} - a_k$ is the same value every time. Therefore, this is an AP with common difference as 4 and first term as 7.

Now, we know, the sum of nth term is;
$S_n = n/2\ [2a + (n - 1)d]$
$S_{15} = 15/2\ [2(7) + (15 - 1) \times 4]$
$= 15/2\ [(14) + 56]$
$= 15/2\ (70)$
$= 15 \times 35$
$= 525$

(ii) $a_n = 9 - 5n$ $a_1 = 9 - 5 \times$
$1 = 9 - 5 = 4$ $a_2 = 9 - 5 \times 2$
$= 9 - 10 = -1$ $a_3 = 9 - 5 \times 3$
$= 9 - 15 = -6$
$a_4 = 9 - 5 \times 4 = 9 - 20 = -11$

We can see here, the common difference between the terms are;

$a_2 - a_1 = -1 - 4 = -5$ a_3
$- a_2 = -6 - (-1) = -5$
$a_4 - a_3 = -11 - (-6) = -5$

Hence, $a_{k+1} - a_k$ is same every time. Therefore, this is an A.P. with common difference as -5 and first term as 4.

Now, we know, the sum of nth term is;
$S_n = n/2\ [2a + (n - 1)d]$
$S_{15} = 15/2\ [2(4) + (15 - 1)(-5)]$
$= 15/2\ [8 + 14(-5)]$
$= 15/2\ (8 - 70)$
$= 15/2\ (-62)$
$= 15(-31)$
$= -465$

11. If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly find the 3rd, the10th and the nth terms.

Solution: Given that,
$S_n = 4n - n^2$
First term, $a = S_1 = 4(1) - (1)^2 = 4 - 1 = 3$
Sum of first two terms = $S_2 = 4(2) - (2)^2 = 8 - 4 = 4$
Second term, $a_2 = S_2 - S_1 = 4 - 3 = 1$
Common difference, $d = a_2 - a = 1 - 3 = -2$

Nth term, $a_n = a + (n - 1)d$
$= 3 + (n - 1)(-2)$
$= 3 - 2n + 2$
$= 5 - 2n$

Therefore, $a_3 = 5 - 2(3) = 5 - 6 = -1$
$a_{10} = 5 - 2(10) = 5 - 20 = -15$

Hence, the sum of first two terms is 4. The second term is 1.
The 3rd, the 10th, and the nth terms are -1, -15, and $5 - 2n$ respectively.

12. Find the sum of first 40 positive integers divisible by 6.

Solution: The positive integers that are divisible by 6 are 6, 12, 18, 24 ….
We can see here, that this series forms an A.P. whose first term is 6 and common difference is 6.
$a = 6$ $d = 6$
$S_{40} = ?$

By the formula of sum of nth term, we know,
$S_n = n/2\ [2a + (n - 1)d]$
Therefore, putting n = 40, we get,
$S_{40} = 40/2\ [2(6) + (40 - 1)\ 6]$
$= 20[12 + (39)\ (6)]$
$= 20(12 + 234)$
$= 20 \times 246$
$= 4920$

13. Find the sum of first 15 multiples of 8.

Solution: The multiples of 8 are 8, 16, 24, 32…
The series is in the form of AP, having first term as 8 and common difference as 8.
Therefore, $a = 8$
$d = 8$
$S_{15} = ?$

By the formula of sum of nth term, we know,
$S_n = n/2\ [2a + (n - 1)d]$
$S_{15} = 15/2\ [2(8) + (15 - 1)8]$
$= 15/2[6 + (14)\ (8)]$
$= 15/2[16 + 112]$
$= 15(128)/2$
$= 15 \times 64$
$= 960$

14. Find the sum of the odd numbers between 0 and 50.

Solution: The odd numbers between 0 and 50 are 1, 3, 5, 7, 9 … 49.
Therefore, we can see that these odd numbers are in the form of A.P.
Hence,
First term, $a = 1$
Common difference, $d = 2$
Last term, $l = 49$

By the formulas of last term, we know,

$l = a + (n - 1)\ d$
$49 = 1 + (n - 1)2$
$48 = 2(n - 1)\ n -$
$1 = 24$
$n = 25$ = Number of terms

By the formula of sum of nth term, we know,

$S_n = n/2\ (a + l)$
$S_{25} = 25/2\ (1 + 49)$
$= 25(50)/2$
$=(25)(25)$
$= 625$

15. A contract on construction job specifies a penalty for delay of completion beyond a certain dateas follows: Rs. 200 for the first day, Rs. 250 for the second day, Rs. 300 for the third day, etc., the penalty for each succeeding day being Rs. 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days.

Solution:

We can see, that the given penalties are in the form of A.P. having first term as 200 and common difference as 50.
Therefore, $a = 200$ and $d = 50$

Penalty that has to be paid if contractor has delayed the work by 30 days = S_{30}
By the formula of sum of nth term, we know,
$S_n = n/2\ [2a + (n - 1)d]$

Therefore,
$S_{30}= 30/2\ [2(200) + (30 - 1)\ 50]$
$= 15\ [400 + 1450]$
$= 15\ (1850)$
$= 27750$
Therefore, the contractor has to pay Rs 27750 as penalty.

16. A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs 20 less than its preceding prize, find the value of each of the prizes.

Solution: Let the cost of 1st prize be Rs.P.
Cost of 2nd prize = Rs.$P - 20$

And cost of 3rd prize = Rs.$P - 40$
We can see that the cost of these prizes are in the form of A.P., having common difference as −20 and first term as P.
Thus, $a = P$ *and* $d = -20$

Given that, $S_7 = 700$
By the formula of sum of nth term, we know,
$S_n = n/2\ [2a + (n - 1)d]$

$7/2\ [2a + (7 - 1)d] = 700$
$$\frac{[2a + (6)(-20)]}{2} = 100$$

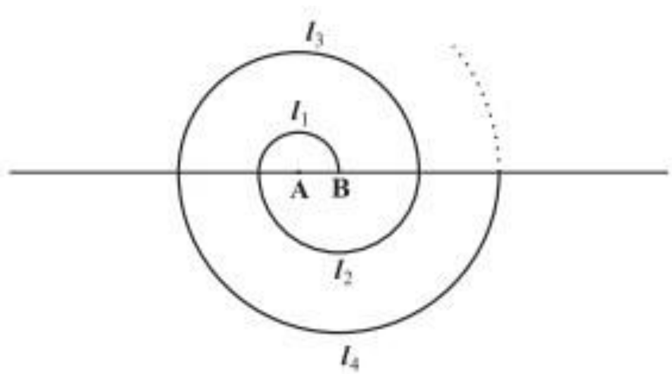
$a + 3(-20) = 100$
$a - 60 = 100$ $a =$
160
Therefore, the value of each of the prizes was Rs 160, Rs 140, Rs 120, Rs 100, Rs 80, Rs 60, and Rs 40.

17. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of class I will plant 1 tree, a section of class II will plant 2 trees and so on till class XII. There are three sections of each class. How many trees will be planted by the students?

Solution:

It can be observed that the number of trees planted by the students is in an AP.
1, 2, 3, 4, 5………………..12
First term, $a = 1$
Common difference, $d = 2 - 1 = 1$
$S_n = n/2\ [2a + (n - 1)d]$
$S_{12} = 12/2\ [2(1) + (12 - 1)(1)]$
$= 6\ (2 + 11)$
$= 6\ (13)$
$= 78$
Therefore, number of trees planted by 1 section of the classes = 78 Number of trees planted by 3 sections of the classes = 3 × 78 = 234 Therefore, 234 trees will be planted by the students.

18. A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A of radii 0.5, 1.0 cm, 1.5 cm, 2.0 cm, ……… as shown in figure. What is the total length of such a spiral made up of thirteen consecutive semicircles? (Take π = 22/7)



Solution: We know,
Perimeter of a semi-circle = πr
Therefore,
$P_1 = \pi(0.5) = \pi/2$ cm
$P_2 = \pi(1) = \pi$ cm P_3 =
$\pi(1.5) = 3\pi/2$ cm
Where, P_1, P_2, P_3 are the lengths of the semi-circles.

Hence we got a series here, as, $\pi/2$,
π, $3\pi/2$, 2π, ...

$P_1 = \pi/2$ cm
$P_2 = \pi$ cm
Common difference, $d = P2 - P1 = \pi - \pi/2 = \pi/2$
First term = $P_1 = a = \pi/2$ cm
By the sum of nth term formula, we know,
$S_n = n/2\, [2a + (n - 1)d]$
Therefor, Sum of the length of 13 consecutive circles is;
$S_{13} = 13/2\, [2(\pi/2) + (13 - 1)\pi/2]$
$= 13/2\, [\pi + 6\pi]$
$= 13/2\, (7\pi)$
$= 13/2 \times 7 \times 22/7$
= 143 cm

19. 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how many rows are the 200 logs placed and how many logs are in the top row?

Solution: We can see that the numbers of logs in rows are in the form of an A.P. 20, 19, 18… For the given A.P.,
First term, $a = 20$ and common difference, $d = a_2 - a_1 = 19 - 20 = -1$

Let a total of 200 logs be placed in n rows.
Thus, $S_n = 200$

By the sum of nth term formula,
$S_n = n/2\ [2a + (n - 1)d]$
$S_{12} = 12/2\ [2(20) + (n - 1)(-1)]$
$400 = n\ (40 - n + 1)$
$400 = n\ (41 - n)\ 400 = 41n$
$- n^2\ n^2 - 41n + 400 = 0\ n^2$
$- 16n - 25n + 400 = 0\ n\ (n$
$- 16) -25\ (n - 16) = 0$
$(n - 16)\ (n - 25) = 0$
Either $(n - 16) = 0$ or $n - 25 = 0\ n$
$= 16$ or $n = 25$

By the nth term formula,
$a_n = a + (n - 1)d\ a_{16} =$
$20 + (16 - 1)\ (-1)\ a_{16} =$
$20 - 15\ a_{16} = 5$
Similarly, the 25th term could be written as;
$a_{25} = 20 + (25 - 1)\ (-1)\ a_{25} = 20 - 24 = -4$
It can be seen, the number of logs in 16th row is 5 as the numbers cannot be negative.
Therefore, 200 logs can be placed in 16 rows and the number of logs in the 16th row is 5.

20. In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato and other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line.

A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?
[Hint: to pick up the first potato and the second potato, the total distance (in metres) run by a competitor is 2 × 5 + 2 ×(5 + 3)]

Solution: The distances of potatoes from the bucket are 5, 8, 11, 14…, which is in the form of AP.
Given, the distance run by the competitor for collecting these potatoes are two times of the distance at which the potatoes have been kept.
Therefore, distances to be run w.r.t distances of potatoes, could be written as; 10,
16, 22, 28, 34,……….
Hence, the first term, $a = 10$ and $d = 16 - 10 = 6$
S_{10} =?

By the formula of sum of nth term, we know,
$S_{10} = 12/2\ [2(20) + (n - 1)(-1)]$
$= 5[20 + 54]$
$= 5\ (74)$
$= 370$
Therefore, the competitor will run a total distance of 370 m.

Exercise 5.4 Page: 115

1. Which term of the AP : 121, 117, 113, . . ., is its first negative term? [Hint : Find n for an < 0]

Solution: Given the AP series is 121, 117, 113, . . .,
Thus, first term, a = 121
Common difference, d = 117-121= -4

By the nth term formula, a_n
$= a + (n - 1)d$

Therefore,

$a_n = 121 + (n - 1)(-4)$
$= 121-4n + 4$
$=125-4n$

To find the first negative term of the series, $a_n < 0$
Therefore,
$125-4n < 0$ 125
$< 4n$ $n>125/4$
$n>31.25$

Therefore, the first negative term of the series is 32nd term.

2. The sum of the third and the seventh terms of an AP is 6 and their product is 8. Find the sum of first sixteen terms of the AP.

Solution: From the given statements, we can write,
$a_3 + a_7 = 6$ ……………………………….(i)
And $a_3 \times a_7 = 8$
……………………………..(ii)

By the nth term formula, a_n
$= a + (n - 1)d$

Third term, $a_3 = a + (3 - 1)d$
$a_3 = a + 2d$……………………………….(iii)

And Seventh term, $a_7 = a + (7 - 1)d$
$a_7 = a + 6d$ ………………………………..(iv)

From equation (iii) and (iv), putting in equation(i), we get,
$a + 2d + a + 6d = 6$ $2a + 8d = 6$
$a+4d=3$
or
$a = 3 – 4d$ ………………………………….(v)

Again putting the eq. (iii) and (iv), in eq. (ii), we get,

$(a + 2d) \times (a + 6d) = 8$

Putting the value of a from equation (v), we get,

$(3 – 4d + 2d) \times (3 – 4d + 6d) = 8$

$(3 – 2d) \times (3 + 2d) = 8$

$3^2 – 2d^2 = 8$
$9 – 4d^2 = 8$ $4d^2$
$= 1$ $d = 1/2$ or
$-1/2$

Now, by putting both the values of d, we get,

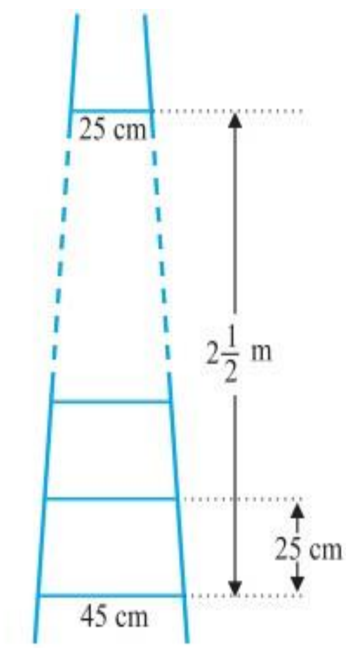
a = 3 – 4d = 3 – 4(1/2) = 3 – 2 = 1, when d = ½ a
= 3 – 4d = 3 – 4(-1/2) = 3+2 = 5, when d = -1/2

We know, the sum of nth term of AP is;
$S_n = n/2\ [2a + (n - 1)d]$

So, when a = 1 and d=1/2
Then, the sum of first 16 terms are;
$S_{16} = 16/2\ [2 + (16 - 1)1/2] = 8(2+15/2) = 76$

And when a = 5 and d= -1/2
Then, the sum of first 16 terms are;
$S_{16} = 16/2\ [2.5+ (16 - 1)(-1/2)] = 8(5/2)=20$

3. A ladder has rungs 25 cm apart. (see Fig. 5.7). The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and the bottom rungs are $2\frac{1}{2}$m apart, what is the length of the wood required for the rungs? [Hint : Number of rungs = -250/25].



Solution: Given,
Distance between the rungs of the ladder is 25cm.
Distance between the top rung and bottom rung of the ladder is=
= 250cm

Therefore, total number of rungs = 250/25 + 1 = 11

$2\frac{1}{2}\text{m} = 2\frac{1}{2} \times 100 \text{ cm} = \frac{5}{2} \times 100 \text{cm}$

As we can see from the figure, the ladder has rungs in decreasing order from top to bottom. Thus, we can conclude now, that the rungs are decreasing in an order of AP.

And the length of the wood required for the rungs will be equal to the sum of the terms of AP series formed. So,
First term, a = 45
Last term, l = 25
Number of terms, n = 11

Now, as we know, sum of nth terms is equal to,
$S_n = n/2(a+ l)$

$S_n = 11/2(45+25) = 11/2 (70) = 385$ cm

Hence, the length of the wood required for the rungs is 385cm.

4. The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x. [Hint : Sx – 1 = S49 – Sx]

Solution: Given,
Row houses are numbers from 1,2,3,4,5…….49.
Thus we can see the houses numbered in a row are in the form of AP.
So,
First term, a = 1
Common difference, d=1

Let us say the number of xth houses can be represented as;

Sum of nth term of AP = n/2[2a+(n-1)d]

Sum of number of houses beyond x house = S_{x-1}
= (x-1)/2[2.1+(x-1-1)1]

= (x-1)/2 [2+x-2]

$= \frac{[x(x-1)]}{2}$ … …………………………………………(i)

By the given condition, we can write,

$S_{49} - S_x$ = {49/2[2.1+(49-1)1]} – {x/2[2.1+(x-1)1]}

= 25(49) – x(x + 1)/2 ………………………………….(ii)

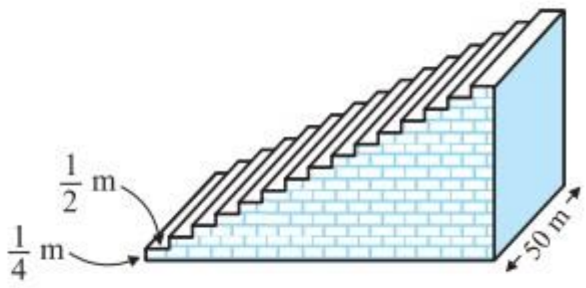
As per the given condition, eq.(i) and eq(ii) are equal to each other; Therefore,

$$\frac{[x(x-1)]}{2} = 25(49) - \frac{x(x+1)}{2}$$

$$x = \pm 35$$

As we know, the number of house cannot be an a negative number. Hence, the value of x is 35.

5. A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete. Each step has a rise of 1 4 m and a tread of 1 2 m. (see Fig. 5.8). Calculate the total volume of concrete required to build the terrace. [Hint : Volume of concrete required to build the first step =$\frac{1}{4} \times \frac{1}{2} \times$ 50 m^3].



Solution: As we can see from the given figure, the first step is ½ m wide, 2nd step is 1m wide and 3rd step is 3/2m wide. Thus we can understand that the width of step by ½ m each time when height is ¼ m. And also, given length of the steps is 50m all the time. So, the width of steps forms a series AP in such a way that;

½ , 1, 3/2, 2, ……..

Volume of steps = Volume of Cuboid

= Length × Breadth × Height

Now,

Volume of concrete required to build the first step = $\frac{1}{4} \times \frac{1}{2} \times 50 = \frac{25}{4}$

Volume of concrete required to build the second step = $\frac{1}{4} \times 1 \times 50 = \frac{25}{2}$

Volume of concrete required to build the second step = $\frac{1}{4} \times \frac{3}{2} \times 50 = \frac{75}{2}$

Now, we can see the volumes of concrete required to build the steps, are in AP series;
$\frac{25}{4}, \frac{25}{2}, \frac{75}{2} \ldots \ldots \ldots$.

Thus, applying the AP series concept, First
term, a = $\frac{25}{4}$
Common difference, d = $\frac{25}{2} - \frac{25}{4} = \frac{25}{4}$

As we know, the sum of nth term is;
$S_n = n/2[2a+(n-1)d] = \frac{15}{2}[2.\frac{25}{4} + (\frac{15}{2} - 1)\frac{25}{4}]$
Upon solving, we get,
$S_n = \frac{15}{2}(100)$
$S_n = 750$

Hence, the total volume of concrete required to build the terrace is 750 m^3.