NCERT Solution For Class 11 Physics Chapter 13 Kinetic Theory of Gases

Q.1: Estimate the fraction of molecular volume to the actual volume occupied by oxygen gas at STP. Take the diameter of an oxygen molecule to be 3 Å

Sol:

Diameter of an oxygen molecule, **d = 3Å**
Radius, r =d/2 = 1.5 Å **= 1.5 × 10^{-8} cm**
We know:
Actual volume occupied by 1 mole of oxygen at **STP = 22400 cm^3**
Molecular volume of oxygen, **V = $N_A(4\pi r^3/3)$**
Where, **N** is **Avogadro's number = 6.023 × 10^{23} molecules/mole**
Therefore, molecular volume of oxygen ,V = 6.023 × 10^{23} x 3.14 (1.5 × $10^{-8})^2$ x (4/3) = **8.51 cm^3**
Thus, ratio of the molecular volume to the actual volume of oxygen = 8.51/22400 = 3.8 x 10^{-4}

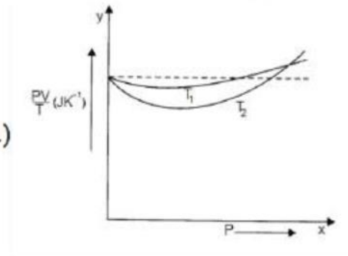
Q.2: Molar volume is the volume occupied by 1 mol of any (ideal) gas at standard temperature and pressure (STP : 1 atmospheric pressure, 0 °C). Show that it is 22.4 litres

Ans:

We know that the ideal gas equation: PV = nRT
Where, R is the universal gas constant = 8.314 J mol^{-1} K^{-1}
n = Number of moles = 1
T = Standard temperature = 273 K
P = Standard pressure = 1 atm = 1.013 × 10^5 Nm^{-2}
Thus, V = (nRT)/p
= (1 x 8.314 x 273)/(1.013 × 10^5)
= 0.0224 m^3
= 22.4 liters
Thus, it is proved that molar volume of a gas at standard temperature and pressure is 22.4 liters.

Q.3. Figure 13.8 shows plot of PV/T versus P for 1.00×10−3 kg of oxygen gas at two different temperatures.

(a) What does the dotted plot signify?
(b) Which is true: T1 > T2 or T1 < T2?
(c) What is the value of PV/T where the curves meet on the y-axis?
(d) If we obtained similar plots for 1.00×10−3 kg of hydrogen, would we get the same value of PV/T at the point where the curves meet on the y-axis? If not, what mass of hydrogen yields the same value of PV/T (for low pressure high temperature region of the plot) ? (Molecular mass of H2 = 2.02 u, of O2 = 32.0 u,R = 8.31 J mo1−1 K−1.)



Sol:

(i) The dotted plot signifies the ideal gas behaviour of oxygen as it is parallel to P −axis and it says that the ratio PV/T remains constant even when P is changed.

(ii) The dotted line in the plot stands for an ideal gas. At temperature T_1 the curve of the gas is closer to the dotted plot than at temperature T_2. A real gas behaves more like an ideal gas when its temperature rises. Thus, T1 > T2 is true for the given graph.

(iii) At the point where the curves meet PV/T = μR
Where μ = no. of moles = 1/32
R = 8.314 J mol^{-1} K^{-1}
Thus, PV/T = (1/32) x 8.314 = **0.26 J K^{-1}**
(iv) Even if we obtain a similar curve for 1 x 10^{-3} kg of hydrogen, we will not get the same value for PV/T because the molar mass of H_2 is 2.02 u and not 32u.
We have:
PV / T = 0.26
Given:
Molecular mass of hydrogen , M = 2.02 u
PV/T = μ R
where, μ = m/M
i.e. PV/T = R (m/M)
i.e. m = MVP/ TR
= 0.26 x (2.02/8.31) = **6.3 x 10^{-5} kg**
Hence, 6.3 x 10^{-5} kg of H_2 will give the value of PV/T = 0.26 J K^{-1}
Q.4: A 30 liters oxygen cylinder has an initial temperature and gauge pressure of 27 ^{0}C and 20 atm respectively. When a certain amount of oxygen escapes from the cylinder the temperature and gauge pressure drops to 17 ^{0}C and 22 atm, respectively. Find the mass of oxygen that escaped the cylinder. [R = 8.31 J mol^{-1} K^{-1}, molecular mass of O_2 = 32 u]

Sol:

Given:
Initial volume of oxygen, V_1 = 30 liters = **30 × 10^{-3} m^3**
Gauge pressure, P_1 = 30, atm = **30 × 1.013 × 10^5 Pa**
Temperature, T_1 = 27°C = **300 K**
Universal gas constant, **R = 8.314 J $mole^{-1}$ K^{-1}**
Let the initial number of moles of oxygen in the cylinder be n_1 .
We know:
$P_1 V_1 = n_1RT_1$
i.e. $n_1 = P_1 V_1 / RT_1$
= (30.39 × 10^5 × 30 × 10^{-3}) / (8.314 × 300)
= 36.552
But, $n_1 = m_1/ M$
Where,
m_1 = initial mass of oxygen
M = molecular mass of oxygen = 32 g
i.e. $m_1 = n_1$ x M = 36.552 x 32 = 1169.6 g
After some oxygen escapes:
Volume, V_2 = 30 x 10^{-3} m^3
Gauge pressure, P_2 = 22; atm = 22 × 1.013 × 10^5 Pa
Temperature, T_2 = 17°C = **290 K**
Let the number of moles of oxygen left in the cylinder be n_2.
Now:
$P_2 V_2 = n_2 RT_2$
i.e. $n_2 = P_2 V_2 / RT_2$
= (22.286 × 10^5 × 30 × 10^{-3})/(8.314 × 290) = 27.72
But, **$n_2 = m_2 / M$**
Where, m_2 = remaining mass of oxygen
i.e. $m_2 = n_2$ x M = 27.72 x 32 = **906.2g**
Therefore the mass of oxygen that escaped the cylinder = $m_1 - m_2$ = 1169.6 – 906.2 = 263.4 g

Q.5: An air bubble of volume 1.0 cm^3 rises from the bottom of a lake 40 m deep at a temperature of 12 °C. To what volume does it grow when it reaches the surface, which is at a temperature of 35 °C ?

Sol:

Given:
Volume of the air bubble, V = 2.0 cm^3 **= 1.0 × 10^{-6} m^3**
Bubble ascends a height of , **d = 20 m**
Temperature at a depth of 40 m, T = 12°C **= 285 K**
Temperature at the surface of the lake, T' = 35°C **= 308 K**

The pressure on the surface of the lake: P' = 1 atm = 1 ×1.013 × 10^5 Pa

And, The pressure at the bottom: **P = 1atm + dρg**
Where, **ρ** is the density of water **= 10^3 kg/m^3**
g is the acceleration due to gravity **= 9.8 m / s^2**
i.e. P = 1.013 × 10^5 + 20 × 10^3 × 9.8 = **297300 Pa**
We know:

PV/T = P'V'/T'
Where, V' is the volume of the bubble at the surface.
V' = PVT'/P'T
= (297300 x 2 x 10^{-6} x 308) / (1.013 x 10^5 x 285) **= 6.34 x 10^{-6} m^3 or 6.34 cm^3**
Therefore, the volume of this bubble when it reaches the surface is 6.34 cm^3.

Q.6: Estimate the total number of air molecules (inclusive of oxygen, nitrogen, water vapour and other constituents) in a room of capacity 25.0 m^3 at a temperature of 27 °C and 1 atm pressure.

Sol:

Given:
Volume of the room, V = 25.0 m^3

Temperature of the room, T = 27°C = **300 K**

Pressure in the room, P = 1 atm **= 1 × 1.013 × 10^5 Pa**

According to gas equation:

PV = k_BNT

Where, **k_B is Boltzmann constant = 1.38 × 10^{-23} m^2 kg s^{-2} K^{-1}**
N is the number of air molecules in the room
Now, **N = PV/ k_BT**
= (1.013 x 10^5 x 50) / (1.38 × 10^{-23} x 300) = 1.22 x 10^{27}
Therefore there is 1.22 × 10^{27} molecules in the room.

Q.7: Estimate the average thermal energy of a helium atom at (i) room temperature (27 °C), (ii) the temperature on the surface of the Sun (6000 K), (iii) the temperature of 10 million kelvin (the typical core temperature in the case of a star). Sol:

Given:
(i) At room temperature, T = 27°C = 300 K
Thus, average thermal energy = kT x (3/2)
Where k is Boltzmann constant = 1.38 × 10^{-23} m^2 kg s^{-2} K^{-1}
Thus,
kT x (3/2) = 1.38 × 10^{-23} x 300 x 1.5 **= 6.21 x 10^{-21} J**

(ii) In the core of the earth, T = 6150 K
Thus, average thermal energy = kT x (3/2)
i.e. kT x (3/2) = 1.38 × 10^{-19} x 6150 x 1.5 **= 1.27 x 10^{-19} J**
(iii) At core of the sun, T = 10^7
Thus, average thermal energy = kT x (3/2)

i.e. kT x (3/2) = 1.38 × 10^{-19} x 10^7 x 1.5 **= 2.07 x 10^{-16} J**

Q.8: Three vessels of equal capacity have gases at the same temperature and pressure. The first vessel contains neon (monatomic), the second contains chlorine (diatomic), and the third contains uranium

hexafluoride (polyatomic). Do the vessels contain an equal number of respective molecules? Is the root mean square speed of molecules the same in the three cases? If not, in which case is Vrms the largest?

Sol:

According to **Avogadro's principle**, gases of the same volume at the same values of temperature and pressure will contain the same number of molecules. Thus, in the above case all the containers will contain **equal number of molecules.**
For a gas of mass (m) at temperature (T), its root mean square speed;

$V_{rms} = \sqrt{\frac{3kT}{m}}$

Where k is the **Boltzmann constant**.
As **k and T are constants**, we get:

$V_{rms} = \sqrt{\frac{1}{m}}$

Thus, V_{rms} is not the same for the molecules of the three gases.
As mass of neon is the least, it will have the highest V_{rms}.

Q.9: At what temperature is the root mean square speed of an atom in an argon gas cylinder equal to the rms speed of a helium gas atom at − 20 °C ? (atomic mass of Ar = 39.9 u, of He = 4.0 u)

Sol:

Given:
Temperature of the **helium atom, T' =** −20°C = **253 K**
Atomic mass of **argon, M = 39.9 u**
Atomic mass of **helium, M' = 4.0 u**
Let, (V_{RMS}) Ar be the rms speed of argon and (V_{RMS}) He be the rms speed of helium.
Now, we know:

$(V_{RMS})Ar = \sqrt{\frac{3RT}{M}}$ . . . . (1)

Where, **R** is the **universal gas constant** and **T** is **temperature of argon** gas

Now, $(V_{RMS})He = \sqrt{\frac{3RT'}{M'}}$ . . . . (2)

According to the question
$(V_{RMS})Ar = (V_{RMS})He$

i e. $\sqrt{\frac{3RT}{M}} = \sqrt{\frac{3RT'}{M'}}$

i.e. T/M = T' / M'
T = M × (T'/M')
Therefore the temperature of argon, T = 39.9 × 253/ 4= 2.52 × 10^3 K

Q.10: Estimate the mean free path and collision frequency of a nitrogen molecule in a cylinder containing nitrogen at 2.0 atm and temperature 17 ^{0}C. Take the radius of a nitrogen molecule to be roughly 1.0 Å. Compare the collision time with the time the molecule moves freely between two successive collisions (Molecular mass of N2 = 28.0 u)

Sol:

Given:
Pressure inside the cylinder containing **nitrogen,** P = 1.0 atm = **1 × 1.013 × 10^5 Pa**

Temperature inside the cylinder, T = 17°C = 290 K
Radius of a nitrogen molecule, r = 1.0 Å = **1 × 1010 m**
Diameter, d = 2 × 1 × 1010 = **2 × 10^{-10} m**
Molecular mass of nitrogen, M = 28.0 g = **28 × 10^{-3} kg**

We know, the root mean square speed ,$V_{RMS} = \sqrt{\frac{3RT}{M}}$

$$V_{RMS} = \sqrt{\frac{3\times 8.314\times 290}{28\times 10^{-3}}} = 508.26 \text{ m/s}$$

For the mean free path (l) we have:

$l = \frac{kT}{\sqrt{2} \times \pi \times d^2 \times P}$

Where, k is 1.38 x 10 $^{-23}$ kg m^2 s^{-2} K^{-1}

Therefore, $l = \frac{1.38 \times 10^{-23} \times 290}{\sqrt{2} \times 3.14 \times (2 \times 10^{-10})^2 \times 1.013 \times 10^5}$ = 2.22 x 10^{-7} m

And, **Collision frequency** = V_{RMS} / l = **2.29 x 10 ^{9} s^{-1}**
Collision time T = d / V_{RMS}

= 2 x 10^{-10} / 508.26 = 2.18 x 10^{-10} s = **3.93 x 10^{-13} s**
Time between consecutive collisions:

T' = l / V_{RMS}

= 2.22 x 10^{-7} / 508.26 = **4.36 x 10^{-10} s**
Thus, T'/ T = (4.36 x 10^{-10})/ (2.22 x 10^{-7}) = **1109.41.**
Therefore the time between two consecutive collisions is 1109.41 times the collision time.

Q11: A metre-long narrow bore held horizontally (and closed at one end) contains a 76 cm long mercury thread, which traps a 15 cm column of air. What happens if the tube is held vertically with the open end at the bottom?

Sol:

Length of mercury thread, **l = 76 cm**

Length of the narrow bore, **L = 1 m = 100 cm**

The air column length in between the closed end & mercury, **l_a = 15 cm**

Since the bottom end is open and the bore is vertically held in the air, the air space occupied by the mercury length is: **100 – (76 + 15) = 9 cm**

Hence, total air column length = 20 + 10 = **30 cm**

Let, mercury out flow due to atmospheric pressure be **'h' cm**

Therefore,

The air column length in the bore = **(30 + h) cm**

And, mercury column length **= 80 – h cm**

Initial pressure, **P_1 = 80 cm of mercury**

Initial volume, **V_1 = 20 cm^3**

Final pressure, **P_2 = 80 – (80 – h) = h cm of mercury**

Final volume is **V_2 = (30 + h) cm^3**

Throughout the process the temperature is constant.

$$P_1V_1 = P_2V_2$$

$$70 \times 20 = h(30 + h)$$

$$h^2 + 30h – 1400 = 0$$

Therefore, $h = \frac{-30 \pm \sqrt{(30)^2 + 4 \times 1 \times 1400}}{2 \times 1}$

$$= -55.3\ cm\ or\ 25.3\ cm$$

Height is always positive. **Hence, mercury that flow out from bore is 25.3 cm and mercury that remains in it is 54.7 cm. The air column length is 30 + 25.3 = 55.3 cm**

Q12. From a certain apparatus, the diffusion rate of hydrogen has an average value of 28.7 $cm^3\ s^{-1}$. The diffusion of another gas under the same conditions is measured to have an average rate of 7.2 $cm^3\ s^{-1}$. Identify the gas

[Hint: Graham's law of diffusion states that: $(M_2/\ M_1)^{1/2} = R_1/\ R_2$, where diffusion rates of gas 1 and gas 2 are given by R_1, R_2 and M_1 and M_2 are their molecular masses]

Sol:

Diffusion rate of hydrogen, $R_1 = 30\ cm^3/\ s$

Diffusion rate of the other gas, $R_2 = 8\ cm^3/\ s$

According to Graham's Law of diffusion, we have:

$$\frac{R_1}{R_2} = \sqrt{\frac{M_2}{M_1}}$$

Where, Molecular mass of hydrogen **M_1 = 2.020 g**

Molecular mass of the unknown gas is M_2

Therefore, $M_2 = M_1\left(\frac{R_1}{R_2}\right)^2$

$$= 2.02\left(\frac{30}{8}\right)^2$$

$$= 28.40\ g$$

Nitrogen has the molecular mass 28. Hence the other gas is nitrogen.

Q.13: Throughout the volume of a gas in equilibrium the density and pressure is uniform. It is true only if no external influences are used. Gas column because of gravity doesn't have uniform density or pressure. Density of the gas decreases with height. The dependence precise is given by law of atmosphere $n_2 = n_1 \exp[-mg(h_2 - h_1)/k_BT]$ Where n_1, n_2 are referred to density at h_1 and h_2 respectively. The sedimentation equilibrium equation of liquid column can be derived by using this relation: $n_2 = n_1 exp[-mgN_A(\rho - \rho')(h_2 - h_1)/(\rho RT)]$ Where ρ is the density of the particle suspended, and ρ' is surrounding medium's density. [NA = Avogadro's number & R the universal gas constant.] [To find the suspended particle's apparent weight use Archimedes principle]

Sol:

From law of atmosphere, we have:

$n_2 = n_1 \exp[-mg(h_2 - h_1)/k_BT]$.... **(i)**

Where, at **height h_1, number density is n_1**, and at **height h_2, number density is n_2**

Weight of suspended particle in gas column is mg

Medium density = ρ'

Suspended particle density = ρ

Suspended particle mass = **m'**

Displaced medium's mass = **m**

Suspended particle's volume = **V**

Archimedes' principle states that the weight of the suspended particle in the liquid column is given by

Now, Displaced medium weight – suspended particle weight

$$= mg – m'g$$

$$= mg – V\rho'g = mg – \left(\frac{m}{\rho}\right)\rho'g$$

$$= mg\left(1 – \frac{\rho'}{\rho}\right) \ldots .(ii)$$

$$Gas\ constant,\ R = k_B N$$

$$K_B = \frac{R}{N} \ldots .(iii)$$

$$n_2 = n_1\ exp[-mg(h_2 – h_1)/k_B T]$$

$$= n_1\ exp[-mg\left(1 – \frac{\rho'}{\rho}(h_2 – h_1)\frac{N}{RT\rho}\right)]$$

Q14: Density of some of the solids and liquids are given below. Provide rough estimates of their atom sizes:

Substance	Atomic mass	Density (10^3 kg m^{-3})
Carbon (diamond)	12.01	2.22
Gold	197.00	19.32
Nitrogen	14.01	1.00
Lithium	6.94	0.53
Fluorine (liquid)	19.00	1.14

[Assume in solid and liquid phase the atoms are tightly packed, and use Avogadro's number. Do not take actual numbers obtain for different atomic sizes? Because of tight packing approximation of the crudeness, the range of atomic size in between few Å]

Sol:

Substance	Radius (Å)
Carbon (diamond) Gold	1.29 1.59
Nitrogen (liquid)	1.77
Lithium	1.73
Fluorine (liquid)	1.88

Substance's atomic mass = M

Substance's density = ρ $Avogadro's\ number = N = 6.023 \times 10^{23}$

$$Each\ atom's\ volume = \frac{4}{3}\pi r^3$$

$$N\ number\ of\ molecules'\ volume = \frac{4}{3}\pi r^3 N \ldots .(i)$$

$One\ mole's\ volume = \frac{M}{\rho} \ldots.(iii)$

$\frac{4}{3}\pi r^3 N = \frac{M}{\rho}$

Therefore, r = $\sqrt[3]{\frac{3M}{4\pi\rho N}}$

<u>For carbon:</u>

$M = 12.01 \times 10^{-3}\ kg,$

$\rho = 2.22 \times 10^3 kgm^{-3}$

Therefore, r = $\left(\frac{3\times12.01\times10^{-3}}{4\pi\times2.22\times10^3\times6.023\times10^{23}}\right)^{\frac{1}{3}} = 1.29$

Hence, radius of carbon atom = 1.29 Å

<u>For gold:</u>

$M = 197.01 \times 10^{-3}\ kg,$

$\rho = 19.32 \times 10^3 kgm^{-3}$

Therefore, r = $\left(\frac{3\times197\times10^{-3}}{4\pi\times19.32\times10^3\times6.023\times10^{23}}\right)^{\frac{1}{3}} = 1.59$

Hence, radius of gold atom = 1.59 Å

<u>For nitrogen (liquid):</u>

$M = 14.01 \times 10^{-3}\ kg,$

$\rho = 1.00 \times 10^3 kgm^{-3}$

Therefore, r = $\left(\frac{3\times14.01\times10^{-3}}{4\pi\times1.00\times10^3\times6.23\times10^{23}}\right)^{\frac{1}{3}} = 1.77$

Hence, radius of nitrogen (liquid) atom = 1.77 Å

<u>For lithium:</u>

$M = 6.94 \times 10^{-3}\ kg,$

$\rho = 0.53 \times 10^3 kgm^{-3}$

Therefore, r = $\left(\frac{3\times6.94\times10^{-3}}{4\pi\times0.53\times10^3\times6.23\times10^{23}}\right)^{\frac{1}{3}} = 1.73$

Hence, radius of lithium atom = 1.73 Å

<u>For fluorine (liquid):</u>

$M = 19.00 \times 10^{-3}\ kg,$

$\rho = 1.14 \times 10^3 kgm^{-3}$

Therefore, r = $\left(\frac{3\times19\times10^{-3}}{4\pi\times1.14\times10^3\times6.023\times10^{23}}\right)^{\frac{1}{3}} = 1.88$

Hence, radius of fluorine (liquid) atom = 1.88 Å