NCERT Solution For Class 11 Physics Chapter 7

Q1. Give the location of the centre of mass of a (i) sphere, (ii) cylinder, (iii) ring, and (iv) cube, each of uniform mass density. Does the centre of mass of a body necessarily lie inside the body?

Ans.

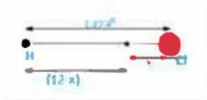
All the given structures are symmetric bodies having a very uniform mass density. Thus, for all the above bodies their center of mass will lie in their geometric centers.

It is not always necessary for a body's center of mass to lie inside it , for example the center of mass of a circular ring is at its center.

Q2. In the HCl molecule, the separation between the nuclei of the two atoms is about 1.27 Å (1 Å = 10-10 m). Find the approximate location of the CM of the molecule, given that a chlorine atom is about 35.5 times as massive as a hydrogen atom and nearly all the mass of an atom is concentrated in its nucleus.

Ans.

Given,



mass of hydrogen atom = 1 unit

mass of chlorine atom = 35.5 unit (As a chlorine atom is 35.5 times the size)

Let the center of mass lie at a distance x from the chlorine atom

Thus, the distance of center of mass from the hydrogen atom = 1.27 – x

Assuming that the center of mass lies of HCL lies at the origin, we get :

x = $\frac{m(1.27-x)+35.5mx}{m-35.5m} = 0$

m(1.27 – x) + 35.5mx = 0

1.27 – x = -35.5x

Therefore, x = (-1.27)/35.5 -1

= -0.037 Å

The negative sign indicates that the center of mass lies at 0.037Å from the chlorine atom.

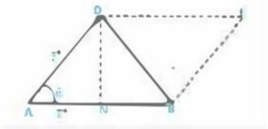
Q3. A child sits stationary at one end of a long trolley moving uniformly with a speed V on a smooth horizontal floor. If the child gets up and runs about on the trolley in any manner, what is the speed of the CM of the (trolley + child) system?

Ans.

The child and the trolley constitute a single system and him moving inside the trolley is a purely internal motion. Since there is no external force on the system the velocity of the center of mass of the system will not change.

Q4. Show that the area of the triangle contained between the vectors a and b is one half of the magnitude of a × b

Ans.



A.

let $\vec{c}$ be presented as $\vec{AB}$ and $\vec{z}$ be represented as $\vec{AD}$, as

represented in the above figure.

Considering ΔADN :

sinθ = DN/AD = DN / $\vec{z}$

DN = $\vec{z}$ sinθ

Now, by definition $|\vec{c} * \vec{z}|$ = $\vec{c} \times \vec{z}$ sinθ

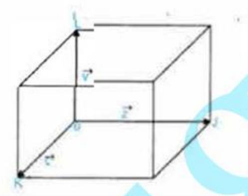
= AB.DN × 2/2 = 2 × area of ABD

Thus, area of ΔADB = ½ × $|\vec{c} * \vec{z}|$

Q5. Show that a.(b × c) is equal in magnitude to the volume of the parallelepiped formed on the three vectors , a, b and c.

Ans.

Let the parallelepiped formed be :



Here, $\vec{OJ} = \vec{z}$, $\vec{OL} = \vec{v}$ *and* $\vec{OK} = \vec{c}$

Now, $\vec{v} * \vec{c}$ = vcsin90^0 $\hat{n}$

Where $\hat{n}$ is a unit vector along OJ perpendicular to the plane

containing $\vec{v}$ *and* $\vec{c}$

Now, $\vec{z}(\vec{v} * \vec{c}) = \vec{z}.v * c\hat{n}$

= z * v * c cos0

= z.(v * c) = Volume of the parallelopiped.

Q6. Find the components along the x, y, z axes of the angular momentum l of a particle, whose position vector is r with

components x, y, z and momentum is p with components px, py and pz. Show that if the particle moves only in the x-y plane the angular momentum has only a z-component.

Ans.

$l_X = yp_Z - zp_Y l_Y$

$= zp_X - xp_Z l_Z$

$= xp_Y - yp_X$

Linear momentum, $\vec{p} = p_x\hat{i} + p_y\hat{j} + p_z\hat{k}$

Position vector of the body, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Angular momentum, $\vec{l} = \vec{r} * \vec{p}$

$= (x\hat{i} + y\hat{j} + z\hat{k}) * (p_x\hat{i} + p_y\hat{j} + p_z\hat{k})$

$= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ p_x & p_y & p_z \end{bmatrix} l_x\hat{i} + l_y\hat{j} + l_z\hat{k} = \hat{i}(yp_z - zp_y) +$

$\hat{j}(zp_x - xp_z) + \hat{k}(xp_y - zp_x)$

From this we can conclude;

$l_X = yp_Z - zp_{y_y}$, $l_Y = zp_X - xp_Z$ and $l_Z = zp_Y - yp_X$

If the body only moves in the x-y plane then $z = p_z = 0$. Which means :

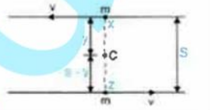
$l_X = l_Y = 0$

And hence only $l_Z = zp_Y - yp_X$, which is just the z component of angular momentum.

Q7. Two particles, each of mass m and speed v, travel in opposite directions along parallel lines separated by a distance d. Show that the angular momentum vector of the two-particle system is the same whatever be the point about which the angular momentum is taken

Ans.

Let us consider three points be Z , C and X :



Angular momentum at Z,

$L_Z = mv \times 0 + mv \times d$

$= mvd$

Angular momentum about B,

$L_X = mv \times d + mv \times 0$

$= mvd$

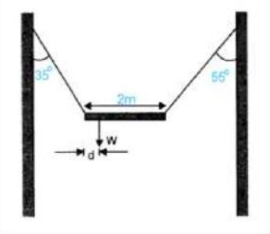
Angular momentum about C,

$L_C = mv \times y + mv \times (s - y) = vdm$

Thus we can see that ;

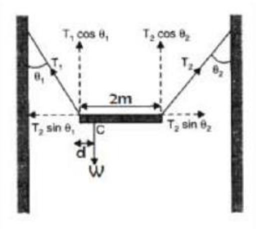
vec{L_Z} = vec{L_X} = vec{L_C} . This proves that the angular momentum of a system does not depend on the point about which its taken.

Q8. A 2m irregular plank weighing W kg is suspended in the manner shown below, by strings of negligible weight. If the strings make an angel of 35^0 and 55^0 respectively with the vertical , find the location of center of gravity of the plank from the left end.



Ans.

The free body diagram of the above figure is:



Given,

Length of the plank, l = 2 m

$\theta_1 = 35^0$ and $\theta_2 = 55^0$

Let T_1 and T_2 be the tensions produced in the left and right strings respectively.

So at translational equilibrium we have;

$T_1 \sin\theta_1 = T_2 \sin\theta_2$

$T_1 / T_2 = \sin\theta_2 / \sin\theta_1 = \sin 55 / \sin 35$

$T_1 / T_2 = 0.819 / 0.573 = 1.42$

$T_1 = 1.42 T_2$

Let 'd' be the distance of center of gravity of the plank from the left.

For rotational equilibrium about the center of gravity :

$T_1 \cos 35 \times d = T_2 \cos 55\ (2 - d)$

$(T_1 / T_2) \times 0.82d = (2 \times 0.57 - 0.57d)$

$1.42 \times 0.82\ d - 0.57\ d = 1.14$

$1.73d = 1.14$

Therefore d = 0.65m

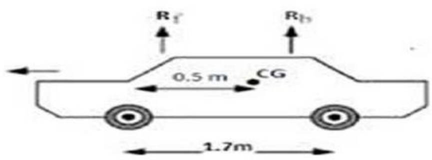
Q9. A car weighs 1800 kg. The distance between its front and back axles is 1.8 m. Its centre of gravity is 1.05 m behind the front axle. Determine the force exerted by the level ground on each front wheel and each back wheel.

Ans.

Given,

Mass of the car, m= 1800 kg

Distance between the two axles, d = 1.8 m
Distance of the C.G. (centre of gravity) from the front axle = 1.05 m
The free body diagram of the car can be drawn as
Let R_F and R_B be are the forces exerted by the level ground on the front and back wheels respectively.

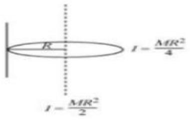


At translational equilibrium:
$R_F + R_B = mg$
$= 1800 \times 9.8$
$= 14700$ N . . . .(1)
For rotational equilibrium about the C.G, we have:
$R_F (0.5) = R_B (1.7 - 0.5)$
$R_B / R_F = 1.2 / 0.5$
$R_B = 2.4 R_F$ . . . . (2)
Using value of equation (2) in equation (1), we get:
$2.4R_F + R_F = 14700$
$R_F = 4323.53$ N
$\therefore R_B = 14700 - 4323.53 = 10376.47$ N
Thus, the force exerted on each of the front wheel = 4323.53/ 2 = 2161.77 N, and
The force exerted on each back wheel = 10376.47 / 2 = 5188.24 N

Q10. (a) Find the moment of inertia of a sphere about a tangent to the sphere, given the moment of inertia of the sphere about any of its diameters to be 2MR2/5, where M is the mass of the sphere and R is the radius of the sphere.
(b) Given the moment of inertia of a disc of mass M and radius R about any of its diameters to be MR2/4, find its moment of inertia about an axis normal to the disc and passing through a point on its edge.

Ans.
Given,
(a) The moment of inertia (M.I .)of a sphere about its radius = $2MR^2/10$

According to the theorem of parallel axes, M.I of a sphere about a tangent to the sphere = $2MR^2/5 + MR^2 = (7MR^2)/5$
(b) Given, moment of inertia of a disc about its diameter = $(MR^2)1/4$
(i)According to the theorem of perpendicular axis, the moment of inertia of a planar body (lamina) about an axis passing through its center and perpendicular to the disc = $2 \times (1/4)MR^2 = MR^2 / 2$



The situation is shown in the given figure.

(ii) Using the theorem of parallel axes:

Moment of inertia about an axis normal to the disc and going through point on its circumference

$= MR^2 / 2 + MR^2$

$= (3MR^2)/2$

Q11. Torques of equal magnitude are applied to a hollow cylinder and a solid sphere, both having the same mass and radius. The cylinder is free to rotate about its standard axis of symmetry, and the sphere is free to rotate about an axis passing through its centre. Which of the two will acquire a greater angular speed after a given time.

Ans.

let m be the mass and r be the radius of the solid sphere and also the hollow cylinder.

Moment of inertia of the solid sphere about an axis passing through its center,

$I_2 = (2mr^2) / 5$

The moment of inertia of the hollow cylinder about its standard axis, $I_1 = mr^2$

Let T be the magnitude of the torque being exerted on the two structures, producing angular accelerations of α_2 and α_1 in the sphere and the cylinder respectively.

Thus we have , $T = I_1\alpha_1 = I_2\alpha_2$

$\therefore \alpha_2 / \alpha_1 = I_1 / I_2 = \frac{mr^2}{\frac{2}{5}mr^2} = 5/2$

$\alpha_2 > \alpha_1$ . . . .(1)

Now, using the relation :

$\omega = \omega_0 + \alpha t$

Where,

ω_0 = Initial angular velocity

t = Time of rotation

ω = Final angular velocity

For equal ω_0 and t, we have:

$\omega \propto \alpha$ . . . . (2)

From equations (1) and (2), we can write:

$\omega_2 > \omega_1$

Thus from the above relation it is clear that the angular velocity of the solid sphere will be greater than that of the hollow cylinder.

Q12. A solid cylinder of mass 20 kg rotates about its axis with angular speed 100 rad s^{-1}. The radius of the cylinder is 0.25 m. What is the kinetic energy associated with the rotation of the cylinder? What is the magnitude of angular momentum of the cylinder about its axis?

Ans.

Given,

Mass of the cylinder, m = 20 kg

Angular speed, ω = 100 rad s^{-1}

Radius of the cylinder, r = 0.25 m

The moment of inertia of the solid cylinder:

$I = mr^2 / 2$

$= (1/2) \times 20 \times (0.25)^2$

$= 0.36 \text{ kg m}^2$
(a) ∴ Kinetic energy = $(1/2) I \omega^2$
$= (1/2) \times 0.36 \times (100)^2 = 1800$ J
(b) ∴ Angular momentum, $L = I\omega$
$= 0.36 \times 100$
$= 36$ Js

Q13. (i) A child stands at the centre of a turntable with his two arms outstretched. The turntable is set rotating with an angular speed of 40 rev/min. How much is the angular speed of the child if he folds his hands back and thereby reduces his moment of inertia to 2/5 times the initial value? Assume that the turntable rotates without friction

(ii) Show that the child's new kinetic energy of rotation is more than the initial kinetic energy of rotation. How do you account for this increase in kinetic energy?

Ans.

(a) Given,

Initial angular velocity, $\omega_1 = 40$ rev/min
let the final angular velocity = ω_2
Let the boy's moment of inertia with hands stretched out = I_1
Let the boy's moment of inertia with hands folded in = I_2
We know :
$I_2 = (2/5) I_1$
As no external forces are acting on the boy, the angular momentum L will be constant.
Thus, we can write:
$I_2 \omega_2 = I_1 \omega_1$
$\omega_2 = (I_1/I_2) \omega_1$
$= [I_1 / (2/5)I_1] \times 40 = (5/2) \times 40 = 100$ rev/min
(b) Final kinetic energy of rotation, $E_F = (1/2) I_2 \omega_2^2$
Initial kinetic energy of rotation, $E_I = (1/2) I_1 \omega_1^2$
$E_F / E_I = (1/2) I_2 \omega_2^2 / (1/2) I_1 \omega_1^2$
$= (2/5) I_1 (100)^2 / I_1 (40)^2$
$= 100$
$\therefore E_F = 100 E_1$
It is clear that there is an increase in the kinetic energy of rotation and it can be attributed to the internal energy used by the boy to fold his hands.

Q14 . A rope of negligible mass is wound round a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder if the rope is pulled with a force of 30 N? What is the linear acceleration of the rope? Assume that there is no slipping.

Ans.

Given,

Mass of the hollow cylinder, m = 3 kg
Radius of the hollow cylinder, r = 40cm = 0.4 m
Force applied, F = 30 N
Moment of inertia of the hollow cylinder about its axis:
$I = mr^2$
$= 2 \times (0.3)^2 = 0.18 \text{ kg m}^2$
Torque, $T = F \times r = 30 \times 0.3 = 9$ Nm
Also, we know that
$T = I\alpha$
(a) Therefore, $\alpha = T / I = 9 / 0.18 = 50 \text{ rad s}^{-2}$
(b) Linear acceleration = $R\alpha = 0.4 \times 50 = 20 \text{ m s}^{-2}$

s-1, an engine needs to transmit a torque of 180 N m. What is the power required by the engine ? (Note: uniform angular velocity in the absence of friction implies zero torque. In practice, applied torque is needed to counter frictional torque). Assume that the engine is 100% efficient.

Ans.

Given,

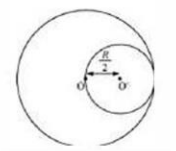
Angular speed of the rotor, ω = 200 rad/s
Torque, T = 180 Nm
Therefore, power of the rotor (P) :

P = T ω
= 200 × 180
= 36.0 kW
Therefore, the engine requires 36.0 kW of power.

Q.16. From a uniform disk of radius R, a circular hole of radius R/2 is cut out. The centre of the hole is at R/2 from the centre of the original disc. Locate the centre of gravity of the resulting flat body

Ans.

Let the mass / unit area of the original disc = σ
Radius of the original disc =2*r*
Mass of the original disc, $m = \pi(2r^2)\sigma = 4\pi r^2\sigma$ . . . . (i)
The disc with the cut portion is shown in the following figure:



Radius of the smaller disc = *r*

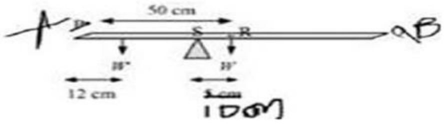
Mass of the smaller disc, $m' = \pi r^2\sigma$

=> m' = m/4 [From equation (i)]
Let O' and O be the respective centers of the disc cut off from the original and the original disc. According to the definition of center of mass, the center of mass of the original disc is concentrated at O, while that of the smaller disc is supposed at O'.
We know that :
OO'= *R*/2 = r2.
After the smaller circle has been cut out, we are left with two systems whose masses are:
−*m'*(= m/4) concentrated at O', *and m* (concentrated at O).
(The negative sign means that this portion has been removed from the original disc.)
Let *X* be the distance of the center of mass from O.
We know :
$X = (m_1 r_1 + m_2 r_2) / (m_1 + m_2)$
$X = [\, m \times 0 - m' \times (r/2)\,] / (\, M + (-M')\,) = -R/6$
(The negative sign indicates that the center of mass is R/6 towards the left of O.)

Q17. A metre stick is balanced on a knife edge at its centre. When two coins, each of mass 5 g are put one on top of the other at the 12.0 cm mark, the stick is found to be balanced at 45.0 cm. What is the mass of the metre stick?

Ans.

The above situation can be represented as :



Let W' and W be the weights of the coin and the meter rule respectively.

The center of mass of the meter rule acts from its center i.e., 45 cm mark

Mass of the meter stick = m'
Mass of each coin, m = 5 g
When the coins are placed 12 cm away from A, the centre of mass shifts by 10 cm from the midpoint R towards A.

The centre of mass is now at 40 cm from A.

For rotational equilibrium about R will
$10 \times g(40 - 12) = m'g(45 - 40)$
$\therefore m' = 38\ g$
Thus, the mass of the meter rule is 38 g.

Q18. A solid sphere rolls down two different inclined planes of the same heights but different angles of inclination. (a) Will it reach the bottom with the same speed in each case? (b) Will it take longer to roll down one plane than the other? (c) If so, which one and why?

Ans.

(a)Let the mass of the ball = m
let the height of the ball = h
let the final velocity of the ball at the bottom of the plane = v

At the top of the plane, the ball possesses Potential energy = mgh

At the bottom of the plane, the ball possesses rotational and translational kinetic energies.
Thus, total kinetic energy = $(1/2)mv^2 + (1/2)I\omega^2$

Using the law of conservation of energy, we have:
$(1/2)mv^2 + (1/2)I\omega^2 = mgh$

For a solid sphere, the moment of inertia about its centre, $I = (2/5)mr^2$
Thus, equation (i) becomes:
$(1/2)mv^2 + (1/2)[(2/5)mr^2]\,\omega^2 = mgh$
$(1/2)\,v^2 + (1/5)\,r^2\omega^2 = gh$
Also , we know $v = r\omega$
$\therefore$ We have : $(1/2)v^2 + (1/5)v^2 = gh$
$v^2\,(7/10) = gh$

$$v = \sqrt{\frac{10}{7}gh}$$

Since the height of both the planes is the same, the final velocity of the ball will also be the same irrespective of which plane it is rolled down.

(b) Let the inclinations of the two planes be θ_1 and θ_2, , where:
$\theta_1 < \theta_2$
The acceleration of the ball as it rolls down the plane with an inclination of θ_1 is:
$g \sin\theta_1$
let R_1 be the normal reaction to the sphere.

Similarly, the acceleration in the ball as it rolls down the plane

ωith an inclination of θ2 is:
g sin θ_2
Let R_2 be the normal reaction to the ball.
Here, $\theta_2 > \theta_1$; sin θ_2 > sin θ_1 . . . . (1)
$\therefore a_2 > a_1$ . . . . (2)
Initial velocity, $u = 0$
Final velocity, v = Constant
Using the first equation of motion:
$v = u + at$
$\therefore t \propto (1/a)$
For inclination θ_1 : $t_1 \propto (1 / a_1)$
For inclination θ_2 : $t_2 \propto (1 / a_2)$

As $a_2 > a_1$ we have :
$t_2 < t_1$
(c)Therefore, the ball will take a greater amount of time to reach the bottom of the inclined plane having the smaller inclination.

Q19. A hoop of radius 2 m weighs 100 kg. It rolls along a horizontal floor so that its centre of mass has a speed of 20 cm/s. How much work has to be done to stop it?

Ans.

Given

Radius of the ring, $r = 2$ m
Mass of the ring, $m = 100$ kg
Velocity of the hoop, v = 20 cm/s = 0.2 m/s
Total energy of the loop = Rotational K.E + Translational K.E..
$E_T = (1/2)mv^2 + (1/2)\, I\omega^2$
We know, the moment of inertia of a ring about its center, $I = mr^2$
$E_T = (1/2)mv^2 + (1/2)\,(mr^2)\omega^2$
Also, we know $v = r\omega$
$\therefore E_T = (1/2)mv^2 + (1/2)mr^2\omega^2$
=> $(1/2)mv^2 + (1/2)mv^2 = mv^2$
Thus the amount of energy required to stop the ring = total energy of the loop.
∴ The amount of work required, $W = mv^2 = 10 \times (0.2)^2 = 0.4$ J.

Q.20. The oxygen molecule has a mass of 5.30 × 10^{-26} kg and a moment of inertia of 1.94×10^{-46} kg m^2 about an axis through its centre perpendicular to the lines joining the two atoms. Suppose the mean speed of such a molecule in a gas is 500 m/s and that its kinetic energy of rotation is two-thirds of its kinetic energy of translation. Find the average angular velocity of the molecule

Ans.

Given,

Mass of one oxygen molecule, $m = 5.30 \times 10^{-26}$ kg

Thus, mass of each oxygen atom = $m/2$

Moment of inertia of jt, $I = 1.94 \times 10^{-46}$ kg m^2
Velocity of the molecule, v = 500 m/s
The distance between the two atoms in the molecule = $2r$

Thus, moment of inertia I, is calculated as:
$I = (m/2)r^2 + (m/2)r^2 = mr^2$
$r = (I/m)^{1/2}$
=> $(1.94 \times 10^{-46} / 5.36 \times 10^{-26})^{1/2} = 0.60 \times 10^{-10}$ m
Given,
$K.E_{rot} = (2/3)K.E_{trans}$
$(1/2)\, I\omega^2 = (2/3) \times (1/2) \times mv^2$
$mr^2\omega^2 = (2/3)mv^2$
Therefore,$\omega = (2/3)^{1/2}\,(v/r)$
$= (2/3)^{1/2}\,(500 / 0.6 \times 10^{-10})$ 750= 4.99×10^{12} rad/s.

Q21. A solid cylinder rolls up an inclined plane of the angle of inclination 30°. At the bottom of the inclined plane, the centre of mass of the cylinder has a speed of 5 m/s.
(a) How far will the cylinder go up the plane?
(b) How long will it take to return to the bottom?

Ans.

Given,

initial velocity of the solid cylinder, v = 5 m/s
Angle of inclination, θ = 30°
Assuming that the cylinder goes upto a height of h.We get :
$(\frac{1}{2}) mv^2 + (\frac{1}{2}) I \omega^2 = mgh$
$(\frac{1}{2}) mv^2 + (\frac{1}{2}) (\frac{1}{2} mr^2) \omega^2 = mgh$
$3/4\ mv^2 = mgh$
$h = 3v^2 / 4g = (3 \times 52) / 4 \times 9.8 = 1.224$ m
let d be the distance the cylinder covers up the plane, this means

$\sin\theta = h/d$
$d = h/\sin\theta = 1.224/\sin 25° = 2.900$m
Now, the time required to return back:

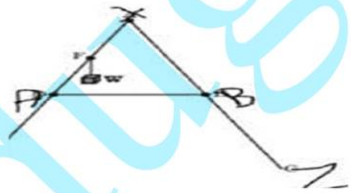
$$t = \frac{2d(1+\frac{K^2}{r^2})}{g\sin\Theta}$$

$$= \sqrt{\frac{2*2.9(1+\frac{1}{2})}{9.8\sin 25°}}$$

= 1.466 s

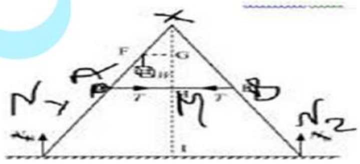
Thus, the cylinder takes 1.466s to return to the bottom.

Q22. As shown in Fig.7.40, the two sides of a step ladder BA and CA are 1.6 m long and hinged at A. A rope DE, 0.5 m is tied half way up. A weight 40 kg is suspended from a point F, 1.2 m from B along with the ladder BA. Assuming the floor to be frictionless and neglecting the weight of the ladder, find the tension in the rope and forces exerted by the floor on the ladder. (Take g = 9.8 m/s2) (Hint: Consider the equilibrium of each side of the ladder separately.)



Ans.

The above situation can drawn as :



Here,

N_Z = Force being applied by floor point Z on the ladder
N_Y = Force being applied by floor point Y on the ladder

T = Tension in string.
YX = XZ = 1.6 m
AB = 0.5 m
YF = 1.2 m
Mass of the weight, m = 30 kg

Now,

Make a perpendicular from X on the floor YZ. This will intersect AB at mid-point M.
ΔXYI and ΔXIZ are similar
∴ZI = IY
This makes I the mid-point of ZY.
AB || ZY
ZY = 2 × AB = 1 m
XF = YX − YF = 0.4 m . . . .(1)
A is the mid-point of XY.
Thus, we can write:
XA = (1/2) × XY = 0.8 m . . . . (2)
Using equations (1) and (2), we get:
FB = 0.4 m
Thus, F is the mid-point of AX.
FG || AM and F is the mid-point of AX. This will make G the mid-point of XM.
ΔFXG and ΔXAM are similar
∴ FG / AM = XF / XA
FG / AM = 0.4 / 0.8 = 1 / 2

FG = (1/2) AM
= (1/2) × 0.25 = 0.125 m

In ΔXAM :
XM = $(XA^2 - AM^2)^{1/2}$
= $(0.8^2 - 0.25^2)^{1/2}$ = 0.76 m

For translational equilibrium of the ladder, the downward force should be equal to the upward force.
$N_Y + N_Z$ = mg = 294 N . . .. (3) [mg = 9.8 x 30]
Rotational equilibrium of the ladder about X is:
$-N_Y \times YI + FG \times mg + N_Z \times ZI - T \times XG + XG \times T = 0$
$-N_Y \times 0.5 + 294 \times 0.125 + N_Z \times 0.5 = 0$
$(N_Z - N_Y) \times 0.5 = 36.75$
$N_Z - N_Y = 73.5$ . . . . (4)
Adding equation (3) and equation (4), we get:
N_Z = 183.75 N
N_Y = 110.25 N
Rotational equilibrium about X for the ladder side XY
$-N_Y \times YI + FG \times mg + T \times XG = 0$
$-110.25 \times 0.5 + 294 \times 0.125 + 0.76 \times T = 0$
∴ T = 24.177 N.

Q23. A man stands on a rotating platform, with his arms stretched horizontally holding a 5 kg weight in each hand. The angular speed of the platform is 30 revolutions per minute. The man then brings his arms close to his body with the distance of each weight from the axis changing from 90cm to 20cm. The moment of inertia of the man together with the platform may be taken to be constant and equal to 7.6 kg m^2.
(a) What is his new angular speed? (Neglect friction.)
(b) Is kinetic energy conserved in the process? If not, from where does the change come about?

Ans.

(a) Given,

Mass of each weight = 5 kg

Moment of inertia of the man-disc system = 7.6 kg m^2

Moment of inertia when his arms are fully stretched to 90 cm:
2 × m r^2
= 2 × 4 × (0.9 $)^2$
= 6.48 kg m^2

Initial moment of inertia of the system, I_i = 6.58 + 5.12 = 11.82 kg m^2

Angular speed, ω_i = 30 rev / min
=> Angular momentum, $L_i = I_i\omega_i$ = 11.82 × 30
= 354.6 . . . . . . (i)
Moment of inertia when he folds his hands inward to 15 cm :
2 × mr^2
= 2 × 4 $(0.15)^2$ = 0.18 kg m^2

Final moment of inertia, I_f = 6.7 + 0.18 = 6.88 kg m^2
let final angular speed = ω_f
=> Final angular momentum, $L_f = I_f\omega_f$ = 6.88 ω_f . . . . . . (ii)
According to the principle of conservation of angular momentum:
$I_i\omega_i = I_f\omega_f$
∴ ω_f = 354.6 / 6.88 = 51.54 rev/min

(b) There is a change in kinetic energy , with the decrease in the moment of inertia kinetic energy increases. The extra kinetic energy is supplied to the system by the work done by the man in folding his arms inside.

Q24. A bullet of mass 10 g and speed 500 m/s is fired into a door and gets embedded exactly at the centre of the door. The door is 1.0 m wide and weighs 12 kg. It is hinged at one end and rotates about a vertical axis practically without friction. Find the angular speed of the door just after the bullet embeds into it. (Hint: The moment of inertia of the door about the vertical axis at one end is ML2/3.)

Ans.

Given, Velocity, v= 500 m/s

Mass of bullet, $m = 10\ g = 10 \times 10^{-3}$ kg

Width of the door, L = 1 m
Radius of the door, $r = 1/2$
Mass of the door, M = 10 kg

Angular momentum imparted by the bullet on the door:
$L = mvr$
$= (8 \times 10^{-3}) \times (450) \times (1/2) = 1.8\ kg\ m^2\ s^{-1}$ ...(i)

Now, Moment of inertia of the door :
$I = ML^2 / 3$
$= (1/3) \times 10 \times 1^2 = 3.33\ kgm^2$
We know, $L = I\omega$
$\therefore \omega = L / I$
$= 1.8 / 3.33 = 0.54$ rad /s

Q25. Two discs of moments of inertia I1 and I2 about their respective axes (normal to the disc and passing through the centre), and rotating with angular speeds ω1 and ω2 are brought into contact face to face with their axes of rotation coincident.

(a) What is the angular speed of the two-disc system?

(b) Show that the kinetic energy of the combined system is less than the sum of the initial kinetic energies of the two discs. How do you account for this loss in energy? Take ω1 ≠ ω2.**Ans.**

(a)Given,

Let the moment of inertia of the two turntables be I_1 and I_2 respectively.

Let the angular speed of the two turntables be ω_1 and ω_2 respectively.

Thus we have;

Angular momentum of turntable 1, $L_1 = I_1\omega_1$
Angular momentum of turntable 2, $L_2 = I_2\omega_2$
=> total initial angular momentum $L_i = I_1\omega_1 + I_2\omega_2$

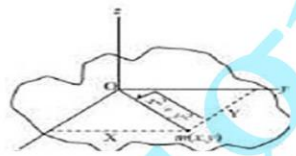
When the two turntables are combined together:
Moment of inertia of the two turntable system, $I = I_1 + I_2$
Let ω be the angular speed of the system.
=> final angular momentum, $L_T = (I_1 + I_2)\ \omega$
According to the principle of conservation of angular momentum, we have:
$L_i = L_T$
$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega$
Therefore , $\omega = (I_1\omega_1 + I_2\omega_2) / (I_1 + I_2)$ . . . . (1)
(b) Kinetic energy of turntable 1, $K.E_1 = (1/2)\ I_1\omega_1^2$
Kinetic energy of turntable 2, $K.E_1 = (1/2)\ I_2\omega_2^2$
Total initial kinetic energy, $KE_I = (1/2)\ (I_1\omega_1^2 + I_2\omega_2^2)$
When the turntables are combined together, their moments of

Angular speed of the system = ω
Final kinetic energy KE_F : = $(1/2)\ (I_1 + I_2)\ \omega^2$
Using the value of ω from (1)
= $(1/2)\ (I_1 + I_2)\ [\ (I_1\omega_1 + I_2\omega_2) / (I_1 + I_2)\ ]^2$
= $(1/2)\ (I_1\omega_1 + I_2\omega_2)^2 / (I_1 + I_2)$
Now, $E_I - E_F$
= $(\ I_1\omega_1{}^2 + I_2\omega_2{}^2)\ (1/2) - [\ (1/2)\ (I_1\omega_1 + I_2\omega_2)^2 / (I_1 + I_2)\ ]$
Solving the above equation, we get
= $I_1\ I_2\ (\omega_1 - \omega_2)^2 / 2(I_1 + I_2)$
As $(\omega_1 - \omega_2)^2$ will only yield a positive quantity and I_1 and I_2 are both positive, the RHS will be positive.
Which means $KE_I - KE_F > 0$
Or, $KE_I > KE_F$
Some of the kinetic energy was lost overcoming the forces of friction when the two turntables were brought in contact.

Q26. (a) Prove the theorem of perpendicular axes. (Hint: Square of the distance of a point (x, y) in the x−y plane from an axis through the origin and perpendicular to the plane is x2+y2).
(b) Prove the theorem of parallel axes. (Hint: If the centre of mass of a system of n particles is chosen to be the origin mi i ∑ r = 0).

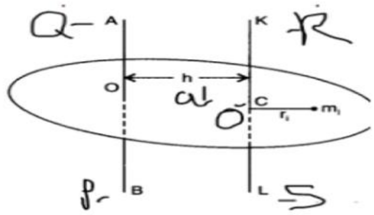
Ans.

(a) According to the theorem of perpendicular axes the moment of inertia of a planar body (lamina) about an axis perpendicular to its plane is equal to the sum of the moments of inertia of the lamina about any two mutually perpendicular axes in its plane and intersecting each other at the point where the perpendicular axis passes through it.
let us consider a physical body with center O and a point mass m,in the x−yplane at (x, y) is shown in the following figure.



Moment of inertia about x-axis, $I_x = mx^2$
Moment of inertia about y-axis, $I_y = my^2$
Moment of inertia about z-axis, $I_z = m(x^2 + y^2)^{1/2}$
$I_x + I_y = mx^2 + my^2$
$= m(x^2 + y^2)$
$= m\ [(x^2 + y^2)^{1/2}]^{1/2}$
$I_x + I_y = I_z$
Thus, the theorem is verified.

(b) According to the theorem of parallel axes the moment of inertia of a body about any axis is equal to the sum of the moment of inertia of the body about a parallel axis passing through its center of mass and the product of its mass and the square of the distance between the two parallel axes.



Suppose a rigid body is made up of n number of particles, having masses $m_1, m_2, m_3, \ldots, m_n$, at perpendicular distances $r_1, r_2, r_3, \ldots, r_n$ respectively from the center of mass O of the rigid body.
The moment of inertia about axis RS passing through the point O:

$$I_{RS} = \sum_{i=1}^{n} m_i r_i^2$$

The perpendicular distance of mass m_i from the axis QP = $a + r_i$

$$I_{QP} = \sum_{i=1}^{n} m_i (a + r_i)^2$$

$$I_{QP} = \sum_{i=1}^{n} m_i (a^2 + r_i^2 + 2ar_i)$$

$$I_{QP} = \sum_{i=1}^{n} m_i a^2 + \sum_{i=1}^{n} m_i r_i^2 + \sum_{i=1}^{n} m_i 2ar_i$$

$$I_{QP} = I_{RS} = \sum_{i=1}^{n} m_i a^2 + 2\sum_{i=1}^{n} m_i a r_i^2$$

We know, the moment of inertia of all particles about the axis passing through the center of mass is zero.

$$2\sum_{i=1}^{n} m_i a r_i = 0$$

as a $\neq 0$

Therefore, $\sum m_i r_i = 0$

Also,

Therefore, $\sum m_i$

= M; M = Total mass of the rigid body

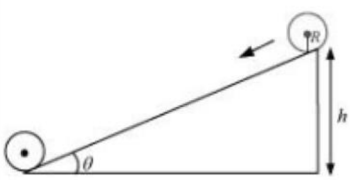
Therefore, $I_{QP} = I_{RS} + Ma^2$

Therefore the theorem is verified.

Q27. Prove the result that the velocity v of translation of a rolling body (like a ring, disc, cylinder or sphere) at the bottom of an inclined plan $v^2 = 2gh/(1+k^2/R^2)$ using dynamical consideration (i.e. by consideration of forces and torques). Note k is the radius of gyration of the body about its symmetry axis, and R is the radius of the body. The body starts from rest at the top of the plane.

Ans.

The above situation can be represented as :



Here,

R = the body's radius

g = Acceleration due to gravity

K = the body's radius of gyration

v = the body's translational velocity

m = Mass of the body

h = Height of the inclined plane

Total energy at the top of the plane, E_T (potential energy) = mgh

Total energy at the bottom of the plane, $E_b = KE_{rot} + KE_{trans}$

$= (1/2)\, I\, \omega^2 + (1/2)\, mv^2$

We know, $I = mk^2$ and $\omega = v / R$

Thus, we have $E_b = \frac{1}{2}mv^2 + \frac{1}{2}(mk^2)(\frac{v^2}{R^2})$

$= \frac{1}{2}mv^2(1 + \frac{k^2}{R^2})$

According to the law of conservation of energy:

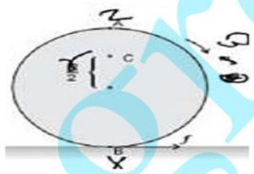
$E_T = E_b$

$mgh = \frac{1}{2}mv^2(1 + \frac{k^2}{R^2})$

$\therefore v = 2gh / [1 + (k^2 / R^2)]$

Thus, the given relation is proved.

Q28. A disc rotating about its axis with angular speed ωo is placed lightly (without any translational push) on a perfectly frictionless table. The radius of the disc is R. What are the linear velocities of the points A, B and C on the disc shown in Fig. 7.41? Will the disc roll in the direction indicated?



Ans.

The respective linear velocities are :

For point A, $v_A = r\omega$ in the direction of the arrow

For point B, $v_B = r\omega$ in the direction opposite to the arrow

For point C, $v_c = (R/2)\omega_o$ in the same direction as that of v_A

Firstly there is no tangential push given to the coin in the initial state. Secondly, the force of friction was the only means of tangential force, but that too is absent as the surface is frictionless. Therefore, the coin cannot roll ahead.

Q29. Explain why friction is necessary to make the disc in Fig. 7.41 roll in the direction indicated.
(a) Give the direction of frictional force at B, and the sense of frictional torque, before perfect rolling begins.
(b) What is the force of friction after perfect rolling begins?

Ans.

(a) Frictional force acts towards right as it opposes the direction of velocity of point X which is towards left. The sense of frictionless torque will be normal to the plane of the coin and outwards.

(b) As force of friction acts in the direction opposite to the velocity at point X, perfect rolling starts only when the force of friction at that point equals zero. This makes the force of friction acting on the coin equal to zero.

Q30. A solid disc and a ring, both of radius 10 cm are placed on a horizontal table simultaneously, with initial angular speed equal to 10 π rad s^{-1}. Which of the two will start to roll earlier? The co-efficient of kinetic friction is μk = 0.2.

Ans.

Given,

Radii of the ring and the disc, r = 10 cm = 0.10 m
Initial angular speed, $\omega_0 = 8\pi$ rad s^{-1}
Coefficient of kinetic friction, $\mu_k = 0.2$
Initial velocity of both the objects, u = 0a

Motion of the two objects is caused by force of friction.
According Newton's second, force of friction, f = ma
$\mu_k mg = ma$
Where,
a = Acceleration produced in the disc and the ring
m = Mass
$\therefore a = \mu_k g$ .... (1)

Using the first equation of motion :
$v = u + at$
$= 0 + \mu_k gt$
$= \mu_k gt$ .... (2)

The frictional force applies a torque in perpendicularly outward direction and reduces the initial angular speed.
Torque, $T = -I\alpha$
Where, α = Angular acceleration
$\mu_k mgr = -I\alpha$
$\therefore \alpha = -\mu_k mgr / I$ .... (3)

According to the first equation of rotational motion, we have :
$\omega = \omega_0 + \alpha t$
$= \omega_0 + (-\mu_k mgr / I)t$ ....(4)
Rolling starts when linear velocity, $v = r\omega$
$\therefore v = r(\omega_0 - \mu_k mgrt / I)$ ...(v)

Using equation (2) and equation (5), we have:
$\mu_k gt = r(\omega_0 - \mu_k mgrt / I)$
$= r\omega_0 - \mu_k mgr^2 t / I$ .... (6)

For the ring:
$I = mr^2$
$\therefore \mu_k gt = r\omega_0 - \mu_k mgr^2 t / mr^2$
$= r\omega_0 - \mu_k gt$
$2\mu_k gt = r\omega_0$
$\therefore t = r\omega_0 / 2\mu_k g$
$= (0.05 \times 8 \times 3.14) / (2 \times 0.2 \times 9.8) = 0.32$ s .... (7)
For the disc: $I = (1/2)mr^2$
$\therefore \mu_k gt = r\omega_0 - \mu_k mgr^2 t / (1/2)mr^2$
$= r\omega_0 - 2\mu_k gt$
$3\mu_k gt = r\omega_0$
$\therefore t = r\omega_0 / 3\mu_k g$
$= (0.05 \times 8 \times 3.14) / (3 \times 0.2 \times 9.8) = 0.213$ s .....(8)

Since $t_D > t_R$, the disc will start rolling before the ring.

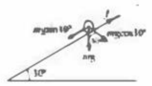
Q31. A cylinder of mass 10 kg and radius 15 cm is rolling

perfectly on a plane of inclination 30o. The coefficient of static friction μs = 0.25.

(a) How much is the force of friction acting on the cylinder?
(b) What is the work done against friction during rolling?
(c) If the inclination θ of the plane is increased, at what value of θ does the cylinder begin to skid,
and not roll perfectly?

Ans.

The above situation can be depicted as:



Given,

mass, m = 10 kg
Radius, r = 15 cm = 0.1 m
Co-efficient of kinetic friction, μ_k = 0.25
Angle of inclination, θ = 25°

We know, moment of inertia of a solid cylinder about its geometric axis, I = $(1/2)mr^2$

The acceleration of the cylinder is given as:

a = mg Sinθ / [m + (I/r^2)]
= mg Sinθ / [m + { ½ mr^2 / r^2}]
= (2/3) g Sin 25°
= (2/3) × 9.8 × = 2.72 ms^{-2}

(a) Using Newton's second law of motion, we can write net force as:
f_{NET} = ma
mg Sin 25° − f = ma
f = mg Sin 25° − ma
= 8 × 9.8 × 0.422 − 10 × 2.72
= 5.88 N

(b) There is no work done against friction during rolling.

(c) We know for rolling without skidding :
μ = (1/3) tan θ
tan θ = 3μ = 3 × 0.25
∴ θ = $\tan^{-1}$ (0.75) = 36.87°

Q32. Read each statement below carefully, and state, with reasons, if it is true or false;
(a) During rolling, the force of friction acts in the same direction as the direction of motion of the CM of the body.
(b) The instantaneous speed of the point of contact during rolling is zero.
(c) The instantaneous acceleration of the point of contact during rolling is zero.
(d) For perfect rolling motion, work done against friction is zero.
(e) A wheel moving down a perfectly frictionless inclined plane will undergo slipping (not rolling) motion.

Ans.

(a) True. This is because during perfect rolling frictional force is zero so work done against it is zero.

(b) True. Rolling occurs only when there is a frictional force to provide the torque so in the absence of friction the wheel simply slips down the plane under the influence of its weight.

(c) True. During rolling the point of the body in contact with the ground does not move ahead (this would be slipping) instead it only touches the ground for an instant and lifts off following a curve. Thus, only if the point of contact remains in touch with the ground and moves forward will the instantaneous speed not be equal to zero.

(d) False. Force of friction acts in the direction opposite to the direction of motion of the centre of mass of the body.

(e) False. The point of contact during rolling as an acceleration in the form of centrifugal force directed towards the centre.

Q33. Separation of Motion of a system of particles into motion of the centre of mass and motion about the centre of mass:

(i) Show $\vec{p_i} = \vec{p'}_i + m_i\vec{V}$

Where p_i is the momentum of the i^{th} particle (of mass m_i) and

$\vec{p_i} + m_i\vec{v'}_i$ ***. Note*** $\vec{v'}_i$ ***is the velocity of the i^{th} particle with respect to the center of mass.***

Also, verify using the definition of the center of mass that

$\sum_i \vec{p_i} = 0$

(ii) Prove that K = K' + ½MV²

Where K is the total kinetic energy of the system of particles, K' is the total kinetic energy of the system when the particle velocities are taken relative to the center of mass and MV^2 /2 is the kinetic energy of the translation of the system as a whole.

(iii) Show $\vec{L} = m_i\vec{L'} + M\vec{R} * \vec{V}$

Where $\vec{L} = \sum \vec{r'}_i * \vec{p'}_i$ ***is the angular momentum of the system about the centre of mass with velocities considered with respect to the centre of mass. Note*** $\vec{r'}_i = \vec{r_i} - \vec{R}$; ***rest of the notation is the standard notation used in the lesson. Remember***

$\vec{L'} and\ \vec{M}R * \vec{v}$ ***can be said to be angular momenta, respectively, about and of the center of mass of the system of particles.***

(iv) Prove that : $\frac{d\vec{l'}}{dt} = \sum \vec{r'}_i * \frac{d\vec{p'}}{dt}$

Further prove that :

$$\frac{d\vec{L'}}{dt} = \vec{T}_{ext}$$

Where $\vec{T_{ext}}$ is the sum of all external torques acting on the

system about the center of mass. (Clue : apply Newton's Third Law and the definition of center of mass . Consider that internal forces between any two particles act along the line connecting the particles.)

Ans.

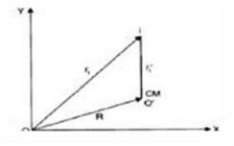
Here $\vec{r_i} = \vec{r'}_i + \vec{R} + R \ldots\ldots$(1)

and, $\vec{V_i} = \vec{V'}_i + \vec{V} \ldots\ldots\ldots$(2)

Where, $\vec{r'}_i$ and $\vec{v'}_i$ represent the radius vector and velocity of

the i^{th} particle referred to center of mass O' as the new origin and

$\vec{V}$ is the velocity of center of mass with respect to O.



(i) Momentum of i^{th} particle

$\vec{p'} \quad = m_i \vec{V'}_i$

= $m_i(\vec{V'}_i + \vec{V})$ [From equation (1)]

Or, $\vec{P} = m_i\vec{V} + \vec{P_i}$

(ii) Kinetic energy of system of particles

K = $\frac{1}{2}\sum m_i V_i^2$

= $\frac{1}{2}\sum_i m_i \vec{V_i}.\vec{V_i}$

= $\frac{1}{2}\sum_i m_i(\vec{V'}_i + \vec{V})(\vec{V'}_i + \vec{V})$

= $\frac{1}{2}\sum_i m_i(\vec{V'}^2_i + \vec{V}^2 + 2\vec{V'}_i\vec{V})$

= $\frac{1}{2}\sum_i m_i V'^2_i + \frac{1}{2}\sum_i m_i V^2 + \sum_i m_i \vec{V'}_i\vec{V}$

= ½ MV^2 + K'

Where M = $\sum m_i$ = total mass of the system.

K' = $\frac{1}{2}\sum m_i V'^2_i$

= kinetic energy of motion about the center of mass.

Or, ½ Mv^2 = kinetic energy of motion of center of mass.(Proved)

Since, $\sum_i m_i \vec{V'}_i \vec{V}$ = $\sum \ m_i \frac{d\vec{r}_i}{dt}\vec{V}$

=0

(iii) Total angular momentum of the system of particles.

$\vec{L} = \vec{r}_i * \vec{p}$

= $(\vec{r}_i + \vec{R}) * \sum_i m_i(\vec{V'}_i + \vec{V})$

= $\sum_i(\vec{R} * m_i\vec{V}) + \sum_i(\vec{r'}_i * m_i\vec{V'}_i) + (\sum_i m_i\vec{r'}_i) *$

$\vec{V} + \vec{R} * \sum_i m_i\vec{V}_i$

= $\sum_i(\vec{R} * m_i\vec{V}) + \sum_i(\vec{r'}_i * m_i\vec{V'}_i) + (\sum_i m_i\vec{r'}_i) *$

$\vec{V} + \vec{R} * \frac{d}{dt}(\sum_i m_i\vec{r'}_i)$

However, we know $\sum_i m_i\vec{r'}_i$ = 0

Since, $\sum_i m_i\vec{r'}_i$ = $\sum_i m_i(\vec{r'}_i - \vec{R}) = M\vec{R} - M\vec{R}$ = 0

According to the definition of center of mass,

$\sum_i(\vec{R} * m_i\vec{V}) = \vec{R} * M\vec{V}$

Such that, $\vec{L} = \vec{R} * M\vec{V} + \sum_i \vec{r'}_i * \vec{P}_i$

Given, $\vec{L} = \sum_i \vec{r'}_i * \vec{p'}_i$

Thus, we have ; $\vec{L} = \vec{R} * M\vec{V} + \vec{L'}$

(iv) From previous solution

$\vec{L'} = \sum_i \vec{r}_i * \vec{P}_i \ \frac{d\vec{L'}}{dt} = \sum_i \vec{r}_i * \frac{d\vec{P}_i}{dt} + \sum_i \frac{d\vec{r'}_i}{dt} * \vec{P}_i$

= $\sum_i \vec{r'}_i * \frac{d\vec{P'}_i}{dt}$

= $\sum \vec{r'}_i * \ \vec{F}_i^{ext}$ = $\vec{T'}_{ext}$

Since, $\sum \frac{d\vec{r'}_i}{dt} * \vec{P}_i \quad \sum \frac{d\vec{r'}_i}{dt} * m\vec{v}_i$ = 0

Total torque = $\vec{T'}_{ext} \quad \sum \vec{r'}_i * \vec{F}_i^{ext}$

$$\sum(\vec{r'}_i + \vec{R}) * \vec{F}_i^{ext}$$

= $\vec{T'}_{ext} + \vec{T}_0^{(ext)}$

Where, $\vec{T'}_{ext}$ is the net torque about the center of mass as

origin and $\vec{T}_0^{ext}$ is about the origin O.

$$\sum \vec{r'}_i * \vec{F}_i^{ext}$$

= $\sum \vec{r'}_i * \frac{d\vec{P'}_i}{dt}$

= $\frac{d}{dt} \sum(\vec{r'}_i * \vec{P'}_i)$ = $\frac{d\vec{L'}}{dt}$

Thus we have , $\frac{d\vec{L'}}{dt}$ = $\vec{T'}_{ext}$