NCERT Solution For Class 11 Physics Chapter 9 Mechanical Properties Of Solids

Q1. A steel wire of length 4.7 m and cross-sectional area 3.0×10^{-5} m^2 stretches by the same amount as a copper wire of length 3.5 m and cross-sectional area of 4.0×10^{-5} m^2 under a given load. What is the ratio of the Young's modulus of steel to that of copper?

Ans.

Given,

Length of the steel wire, L_1 = 5 m
The cross-sectional area of the steel wire, $A_1 = 3.0 \times 10^{-5}$ m^2
Length of the copper wire, L_2 = 4 m
Cross-sectional area of the copper wire, $A_2 = 4.0 \times 10^{-5}$ m^2

Change in length = $\Delta L_1 = \Delta L_2 = \Delta L$
Let the force being applied in both the situations = F

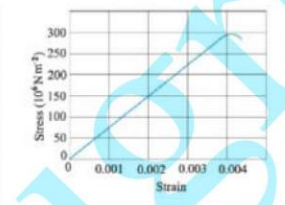
We know, Young's modulus of the steel wire :
$Y_1 = (F_1 / A_1) (L_1 / \Delta L_1)$
$= (F / 3 X 10^{-5}) (5 / \Delta L)$ .... (1)

Also, Young's modulus of the copper wire:
$Y_2 = (F_2 / A_2) (L_2 / \Delta L_2)$
$= (F / 4 \times 10^{-5}) (3 / \Delta L)$ ....(2)

Dividing equation (1) by equation (2), we get :
$Y_1 / Y_2 = (5 \times 4 \times 10^{-5}) / (3 \times 10^{-5} \times 4)$
= 1.6 : 1
The ratio of Young's modulus of steel to Young's modulus of copper is 1.6 : 1

Q2. The figure shows the strain-stress curve for a given material. What are (a) Young's modulus and (b) approximate yield strength for this material?

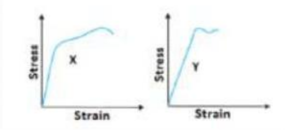
Ans.



(i) It can be seen from the graph that the approximate yield strength of this material is $300 \times 10^6 Nm/^2$or 3×10^8 N/m^2.

(ii) It is observed from the given graph that for strain 0.001, stress is $75 \times 10^6 N/m^2$.
∴ We know, Young's modulus, Y = Stress / Strain
$= 75 \times 10^6 / 0.001 = 7.5 \times 10^{10}$ Nm^{-2}

Q3. The stress-strain graphs for materials X and Y are shown.



The graphs are drawn to the same scale.
(a) Which of the materials has the greater Young's modulus?
(b) Which of the two is the stronger material?

Ans.

(a) Comparing the two graphs we can infer that the stress on X is greater than that on Y for the same values of strain. Therefore, Young's Modulus (stress/strain) is greater for X.

(b) As X's Young's modulus is higher, it is the stronger material among the two. For strength is the measure of stress a material can handle before breaking.

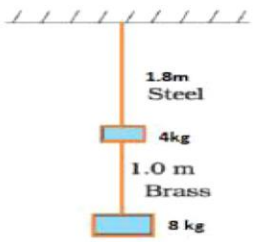
Q4. Read the following two statements below carefully and state, with reasons, if it is true or false.
(a) The Young's modulus of rubber is greater than that of steel;
(b) The stretching of a coil is determined by its shear modulus.Ans.

(a) True. Stretching a coil does not change its length, only its shape is altered and this involves shear modulus.

(b) False. This is because, for the same value of stress, there is more strain in rubber than in steel. And as Young Modulus is an inverse of strain, it is greater in steel.

Q5. Two wires of diameter 0.25 cm, one made of steel and the other made of brass are loaded as shown in the figure. The unloaded length of steel wire is 1.5 m and that of brass wire is 1.0 m. Compute the elongations of the steel and the brass wires.

Ans.



Given,

Diameter of the wire, d = 0.25 m
Hence, the radius of the wires, r = d/2 = 0.125 cm
Length of the steel wire, L_1 = 1.8 m
Length of the brass wire, L_2 = 1.0 m

Total force exerted on the steel wire:

$F_1 = (4 + 8)\,g = 12 \times 9.8 = 117.6$ N
We know, Young's modulus for steel :
$Y_1 = (F_1/A_1) / (\Delta L_1 / L_1)$

Where,
ΔL_1 = Change in the length of the steel wire
A_1 = Area of cross-section of the steel wire = πr_1^2

We know ,Young's modulus of steel, $Y_1 = 2.0 \times 10^{11}$ Pa
$\therefore \Delta L_1 = F_1 \times L_1 / (A_1 \times Y_1)$
$= (117.6 \times 1.8) / [\pi(0.125 \times 10^{-2})^2 \times 2 \times 10^{11}] = 2.15 \times 10^{-4}$m

Total force on the brass wire:
$F_2 = 8 \times 9.8 = 78.4$ N

Young's modulus for brass:

$Y_2 = 0.91 \times 10^{11}$ Pa
Where,
ΔL_2 = Change in the length of the brass wire
A_1 = Area of cross-section of the brass wire = πr_1^2
$\therefore \Delta L_2 = F_2 \times L_2 / (A_2 \times Y_2)$
$= (78.4 \times 1) / [\pi \times (0.125 \times 10^{-2})^2 \times (0.91 \times 10^{11})] = 1.75 \times 10^{-4}$ m

Therefore, Elongation of the steel wire = 2.15×10^{-4}m, and
Elongation of the brass wire = 1.75×10^{-4} m

Q6. The edge of an aluminium cube is 10 cm long. One face of the cube is firmly fixed to a vertical wall. A mass of 100 kg is

then attached to the opposite face of the cube. The shear modulus of aluminium is 25 GPa. What is the vertical deflection of this face?

Ans.

Given,

Shear modulus (η) of aluminium = 25 GPa = 25×10^9 Pa

Edge of the cube, L = 8 cm = 0.08 m

The mass attached to the cube, m = 50 kg

Shear modulus, η = Shear stress / Shear strain = (F/A) / (L/ΔL)

Where,
F = Applied force = mg = 50 × 9.8 = 490 N
A = Area of one face of the cube = 0.08 × 0.08 = 0.0064 m^2
ΔL= Vertical deflection of the cube
∴ ΔL = FL / Aη
= 490 × 0.08 / [0.0064× (25 × 10^9)]
= 2.45 x 10^{-7} m
Therefore the vertical deflection of this face of the cube is 2.45 ×10^{-7} m.

Q7. Four identical hollow cylindrical columns of mild steel support a big structure of mass 50,000 kg. The inner and outer radii of each column are 30 and 60 cm respectively. Assuming the load distribution to be uniform, calculate the compressional strain of each column

Ans.

A.Given,

Mass of the building, M = 50,000 kg

Outer radius of the column, R = 60 cm = 0.6 m

Inner radius of the column, r = 30 cm = 0.3 m

Young's modulus of steel, Y = 2 × 10^{11} Pa

We know,

Total force exerted, F = Mg = 50000 × 9.8 N = 490000 N
Stress = Force exerted on a single column = 490000 / 4 = 122500 N
Also, Young's modulus, Y = Stress / Strain

Strain = (F/A) / Y
Where,
Area, A = π (R^2 – r^2)
= π ($(0.6)^2$ – $(0.4)^2$)
= 0.628
Strain = 61250 / [0.628× 2 × 10^{11}] = 4.87 × 10^{-7}
Therefore, the compressional strain of each column is 4.87 × 10^{-7}.

Q10. A rigid bar of mass 15 kg is supported symmetrically by three wires each 2.0 m long. Those at each end are of copper and the middle one is of iron. Determine the ratios of their diameters if each is to have the same tension.

Ans.

As the tension on the wires is the same, the extension of each wire will also be the same. Now, as the length of the wires is the same, the strain on them will also be equal.

Now, we know :

Y = Stress / Strain
= (F/A) / Strain = ($4F/\pi d^2$) / Strain .... (1)

Where,
A = Area of cross-section
F = Tension force
d = Diameter of the wire
We can conclude from equation (1) that $Y \propto (1/d^2)$
We know that Young's modulus for iron, $Y_1 = 190 \times 10^9$ Pa
Let the diameter of the iron wire = d_1
Also ,Young's modulus for copper, $Y_2 = 120 \times 10^9$ Pa
let the diameter of the copper wire = d_2
Thus, the ratio of their diameters can be given as :

$$\frac{d_1}{d_2} = \sqrt{\frac{Y_1}{Y_2}}$$

$$= \sqrt{\frac{190 \times 10^9}{120x10^9}} = 1:25:1$$

Q11. A 14.5 kg mass, fastened to the end of a steel wire of unstretched length 1.0 m, is whirled in a vertical circle with an angular velocity of 2 rev/s at the bottom of the circle. The cross-sectional area of the wire is 0.065 cm². Calculate the elongation of the wire when the mass is at the lowest point of its path.

Ans.

A.Given,

Mass, m = 14.5kg
Length of the wire, l = 1.0 m
Angular velocity, ω = 2 rev/s = 2 × 2π rad/s = 12.56 rad / s
Cross-sectional area of the wire, a = 0.060 cm^2 = $0.06 \times 10^{-4} m^2$
Let Δl be the increase in the wire's length when the body is at the lower most point.
When the body is at the lowest point of the vertical circle , the force on the body is:
$F = mg + ml\omega^2$
$= 14.5 \times 9.8 + 14.5 \times 1 \times (12.56)^2$
= 2513.304 N
We know, Young's modulus = Stress / Strain
Y = (F/A) / (Δl / l)
∴ Δl = Fl / AY
Also, young's modulus for steel = 2×10^{11} Pa
=> $\Delta l = (2513.304 \times 1) / (0.06 \times 10^{-4} \times 2 \times 10^{11}) = 2.09 \times 10^{-3}$ m
Therefore, the increase in the wire is 2.09×10^{-3} m.

Q12. Compute the bulk modulus of water from the following data: Initial volume = 100.0 litre, Pressure increase = 100.0 atm (1 atm = 1.013 × 10⁵ Pa), Final volume = 100.5 litre. Compare the bulk modulus of water with that of air (at constant temperature). Explain in simple terms why the ratio is so large

Ans.

Given,

P = 100 atmosphere

$= 100 \times 1.013 \times 10^5$ Pa

Final volume, V_2 = 100.5 l = $100.5 \times 10^{-3} m^3$

Initial volume, V_1 = 100.0 l = $100.0 \times 10^{-3} m^3$

Increase in volume, $\Delta V = V_2 - V_1 = 0.5 \times 10^{-3} m^3$

Bulk modulus = $\Delta p / (\Delta V / V_1) = \Delta p \times V_1 / \Delta V$

$= [100 \times 1.013 \times 10^5 \times 100 \times 10^{-3}] / (0.5 \times 10^{-3})$
$= 2.026 \times 10^9$ Pa
We know ,Bulk modulus of air = 1 ×10^5 Pa

∴ Bulk modulus of water / Bulk modulus of air =
$2.026 \times 10^9 / (1 \times 10^5) = 2.026 \times 10^4$
This ratio is very large because air has more intermolecular space thus it is more compressible than water.

Q13. What is the density of water at a depth where pressure is 80.0 atm, given that its density at the surface is 1.03×10^3 kg m^{-3}?

Ans.

let the depth be the alphabet 'd'.

Given,
Pressure at the given depth, $p = 60.0$ atm $= 60 \times 1.01 \times 10^5$ Pa
Density of water at the surface, $\rho_1 = 1.03 \times 10^3$ kg m^{-3}

Let ρ_2 be the density of water at the depth d.
V_1 be the volume of water of mass m at the surface.
Then, let V_2 be the volume of water of mass m at the depth h and ΔV is the change in volume.
$\Delta V = V_1 - V_2$
$= m [(1/\rho_1) - (1/\rho_2)]$
∴ Volumetric strain $= \Delta V / V_1$
$= m [(1/\rho_1) - (1/\rho_2)] \times (\rho_1 / m)$
$\Delta V / V_1 = 1 - (\rho_1/\rho_2)$ . . . . (1)
We know, Bulk modulus, $B = pV_1 / \Delta V$
=> $\Delta V / V_1 = p / B$

Compressibility of water $= (1/B) = 45.8 \times 10^{-11}$ Pa^{-1}
∴ $\Delta V / V_1 = 60 \times 1.013 \times 10^5 \times 45.8 \times 10^{-11} = 2.78 \times 10^{-3}$ . . . . . (2)

Using equation (1) and equation (2), we get:
$1 - (\rho_1/\rho_2) = 2.78 \times 10^{-3}$
$\rho_2 = 1.03 \times 10^3 / [1 - (2.78 \times 10^{-3})]$
$= 1.032 \times 10^3$ kg m^{-3}
Therefore, at the depth d water has a density of 1.034×10^3 kg m^{-3}.

Q14. Compute the fractional change in volume of a glass slab, when subjected to a hydraulic pressure of 10 atm.

Ans.

Given,

Pressure acting on the glass plate, $p = 10$ atm $= 10 \times 1.013 \times 10^5$ Pa

We know,

Bulk modulus of glass, $B = 37 \times 10^9$ Nm^{-2}
=> Bulk modulus, $B = p / (\Delta V/V)$
Where,
$\Delta V/V$ = Fractional change in volume
∴ $\Delta V/V = p / B$
$= [10 \times 1.013 \times 10^5] / (37 \times 10^9)$
$= 2.73 \times 10^{-4}$
Therefore, the fractional change in the volume of the glass plate is 2.73×10^{-4}.

Q15. Determine the volume contraction of a solid copper cube, 10 cm on an edge, when subjected to a hydraulic pressure of 7.0×10^6 Pa.

Ans.

A.Given,

Hydraulic pressure, $p = 7.0 \times 10^6$ Pa

Edge length of the cube, l = 10 cm = 0.10 m

Bulk modulus of copper, B = 140×10^9 Pa

We know, bulk modulus, B = p / ($\Delta V/V$)
Where,
V = Original volume = V = l^3
ΔV = Change in volume
.$\Delta V/V$ = Volumetric strain.
$\Delta V = pV / B$

$\therefore \Delta V = pl^3 / B$
= $[8 \times 10^6 \times (0.05)^3] / (140 \times 10^9)$
= 7.142×10^{-9} m^3 = 7.142×10^{-3} cm^{-3}
Hence, the volume contraction of the solid copper cube is 7.142×10^{-3} cm^{-3}.

Q16. How much should the pressure on a litre of water be changed to compress it by 0.10%?

Ans.

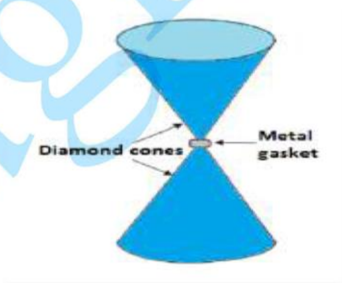
Given, volume of water,V= 1 L
And water needs to be compressed by 0.10%.
$\therefore$ Fractional change, $\Delta V / V = 0.10 / (100 \times 1) = 1.0 \times 10^{-3}$
We know,

Bulk modulus, B = $\rho / (\Delta V/V)$
=> $\rho = B \times (\Delta V/V)$

We know, bulk modulus of water, B = 2.2×10^9 Nm^{-2}
=> $\rho = 2.2 \times 10^9 \times 1.0 \times 10^{-3} = 3.3 \times 10^6$ Nm^{-2}
Thus, a pressure of 3.3×10^6 Nm^{-2} should be applied on the water.

Q17. Anvils made of single crystals of diamond, with the shape as shown in the figure, are used to investigate the behaviour of materials under very high pressures. Flat faces at the narrow end of the anvil have a diameter of 0.50 mm, and the wide ends are subjected to a compressional force of 50,000 N. What is the pressure at the tip of the anvil?

Ans.

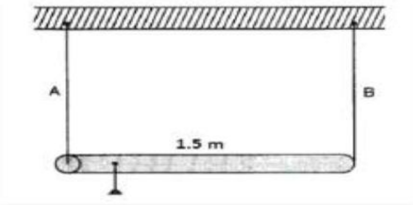


Given,

Diameter at the narrow ends, d = 0.50 mm = 0.5×10^{-3} m
Radius, r = d/2 = 0.25×10^{-3} m
Compressional force, F = 50000 N
Therefore the pressure at the tip of the anvil:
P = Force / Area = $50000 / \pi(0.25 \times 10^{-3})^2$
= 4.07×10^{11} Pa

Q18. A rod of length 1.05 m having negligible mass is supported at its ends by two wires of steel (wire A) and aluminium (wire B) of equal lengths as shown in the figure. The cross-sectional areas of wires A and B are 1.0 mm2 and 2.0 mm2, respectively. At what point along the rod should a mass m be suspended in order to produce (a) equal stresses and (b) equal strains in both steel and aluminium wires.

Ans.

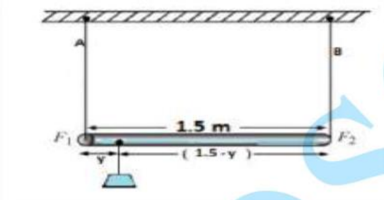


Given,

Cross-sectional area of wire A, $a_1 = 1.0\ mm^2 = 1.0 \times 10^{-6}\ m^2$
Cross-sectional area of wire B, $a_2 = 2\ mm^2 = 2 \times 10^{-6}\ m^2$

We know,Young's modulus for steel, $Y_1 = 2 \times 10^{11}\ Nm^{-2}$
Young's modulus for aluminum, $Y_2 = 7.0 \times 10^{10}\ Nm^{-2}$

(i) Let a mass m be hung on the stick at a distance y from the end where wire A is attached.
Stress in the wire = Force / Area = F / a
Now it is given that the two wires have equal stresses ;
$F_1 / a_1 = F_2 / a_2$
Where,
F_1 = Force acting on wire A
and F_2 = Force acting on wire B
$F_1 / F_2 = a_1 / a_2 = 1 / 2$ . . . . (1)
The above situation can be represented as :



Moment of forces about the point of suspension, we have:
$F_1 y = F_2 (1.5 - y)$
$F_1 / F_2 = (1.5 - y) / y$ . . . . (2)
Using equation (1) and equation (2), we can write:
$(1.5 - y) / y = 1 / 2$
$2 (1.5 - y) = y$
$y = 1$ m

Therefore, the mass needs to be hung at a distance of 1m from the end where wire A is attached in order to produce equal stress in the two wires.

(ii) We know,

Young's modulus = Stress / Strain
=> Strain = Stress / Young's modulus = (F/a)/ Y
It is given that the strain in the two wires is equal :
$(F_1/a_1) / Y_1 = (F_2/a_2) / Y_2$
$F_1 / F_2 = a_1 Y_1 / a_2 Y_2$
$a_1 / a_2 = 1 / 2$
$F_1 / F_2 = (1 / 2)(2 \times 10^{11} / 7 \times 10^{10}) = 10 / 7$ . . . . (3)

Let the mass m be hung on the stick at a distance y_1 from the end where the steel wire is attached in order to produce equal strain

Taking the moment of force about the point where mass m is suspended :

$F_1 y_1 = F_2 (1.5 - y_1)$
$F_1 / F_2 = (1.5 - y_1) / y_1$ . . . . (4)

From equations (3) and (4), we get:

$(1.05 - y_1) / y_1 = 10/7$
$7(1.05 - y_1) = 10y_1$
$y_1 = 0.432$ m

Therefore, the mass needs to be hung at a distance of 0.432 m from the end where wire A is attached in order to produce equal strain in the two wires.

Q19. 9 A mild steel wire of length 1.0 m and cross-sectional area 0.50×10^{-2} cm^2 is stretched, well within its elastic limit, horizontally between two pillars. A mass of 100 g is suspended from the mid-point of the wire. Calculate the depression at the midpoint.

Given,

Water pressure at the bottom, p = 1000 atm = $1000 \times 1.013 \times 10^5$ Pa

$p = 1.01 \times 10^8$ Pa
Initial volume of the steel ball, V = 0.30 m^3

We know, bulk modulus of steel, $B = 1.6 \times 10^{11}$ Nm^{-2}

Let the change in the volume of the ball on reaching the bottom of the trench be ΔV.
Bulk modulus, $B = p / (\Delta V/V)$
$\Delta V = pV / B$
$= [1.01 \times 10^8 \times 0.30] / (1.6 \times 10^{11}) = 1.89 \times 10^{-4}$ m^3
Hence, volume of the ball changes by 1.89×10^{-4} m^3 on reaching the bottom of the trench.

Q20. Two strips of metal are riveted together at their ends by four rivets, each of diameter 6.0 mm. What is the maximum tension that can be exerted by the riveted strip if the shearing stress on the rivet is not to exceed 6.9×10^7 Pa? Assume that each rivet is to carry one-quarter of the load.

Ans.

Given,

Diameter of the metal bar, d = 6.0 mm = 6.0×10^{-3} m
Radius, r = d/2 = 2.5×10^{-3} m

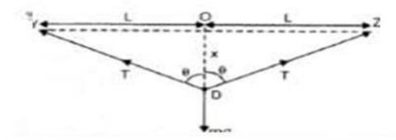
Maximum shearing stress = 6.9×10^7 Pa

We know,

Maximum stress = Maximum force or tension / Area
=> Maximum force = Maximum stress × Area
$= 6.9 \times 10^7 \times \pi \times (2.5)^2$
$= 6.9 \times 10^7 \times \pi \times (2.5 \times 10^{-3})^2$
= 1354.125 N
Since each rivet carries ¼ of the load.
∴ Maximum tension on each rivet = 4 × 1354.125 = 5416.5 N.

Q22. A mild steel wire of cross-sectional area 0.60×10^{-2} cm^2 and length 2 m is stretched (not beyond its elastic limit) horizontally between two columns. If a 100g mass is hung at the midpoint of the wire, find the depression at the midpoint.

Ans.



Let YZ be the mild steel wire of length 2l = 2m and cross sectional area $A = 0.60 \times 10^{-2}$ cm^2. Let the mass of m = 100 g = 0.1 kg be hung from the midpoint O, as shown in the figure.
And

let x be the depression at the midpoint i.e OD

From the figure;

ZO =YO = l = 1 m ;

M = 0.1 KG

ZD = YD = $(l^2 + x^2)^{1/2}$

Increase in length, Δl = YD + DZ − ZY

= 2YD − YZ (As DZ = YD)

= $2(l^2 + x^2)^{1/2} - 2l$

$\Delta l = 2l(x^2/2l^2) = x^2 / l$

Therefore, longitudinal strain = $\Delta l / 2l = x^2/2l^2$ . . . . (i)

If T is the tension in the wires, then in equilibrium $2T\cos\theta = 2mg$

Or, $T = mg / 2\cos\theta$

= $[mg (l^2 + x^2)^{1/2}] / 2x = mgl / 2x$

Therefore, Stress = T / A = mgl / 2Ax . . . . (ii)

$$Y = \frac{stress}{strain} = \frac{mgl}{2Ax} * \frac{2l^2}{x^2}$$

$$x = l\left[\frac{mg}{YA}\right]^{\frac{1}{3}} = 1\left[\frac{0.1*10}{20*10^{11}*0.6*10^{-6}}\right]^{\frac{1}{3}}$$

= 9.41×10^{-3} m.