Exercise 10.1





The vector \overline{OP} represents the displacement of 40 km, 30 east of north. **1. Represent graphically a displacement of 40 km, 30° east of north. Solution:**

2. Classify the following measures as scalars and vectors. (i) 10 kg (ii) 2 metres north-west (iii) 40° (iv) 40 watt (v) 10^{-19} coulomb (vi) 20 m/s² Solution:

- (i) 10 kg is a scalar quantity because it has only magnitude.
- (ii) 2 meters north-west is a vector quantity as it has both magnitude and direction.

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- (iii) 40° is a scalar quantity as it has only magnitude.
- (iv) 40 watts is a scalar quantity as it has only magnitude.
- (v) 10^{-19} coulomb is a scalar quantity as it has only magnitude.
- (vi) 20 m/s² is a vector quantity as it has both magnitude and direction.

3. Classify the following as scalar and vector quantities.

(i) time period (ii) distance (iii) force

(iv) velocity (v) work done Solution:

- (i) Time period is a scalar quantity as it has only magnitude.
- (ii) Distance is a scalar quantity as it has only magnitude.
- (iii) Force is a vector quantity as it has both magnitude and direction.
- (iv) Velocity is a vector quantity as it has both magnitude as well as direction. (v) Work done is a scalar quantity as it has only magnitude.

4. In Figure, identify the following vectors.



(i) Coinitial (ii) Equal (iii) Collinear but not equal Solution:

- (i) Vectors \vec{a} and \vec{d} are coinitial because they have the same initial point.
- (ii) Vectors \vec{b} and \vec{d} are equal because they have the same magnitude and direction.

(iii) Vectors \vec{a} and \vec{c} are collinear but not equal. This is because although they are parallel, their directions are not the same.

5. Answer the following as true or false.

- (i) \vec{a} and $-\vec{a}$ are collinear.
- (ii) Two collinear vectors are always equal in magnitude.
- (iii) Two vectors having same magnitude are collinear.
- (iv) Two collinear vectors having the same magnitude are equal. Solution:

(i) True.

Vectors \vec{a} and $-\vec{a}$ are parallel to the same line.

(ii) False.

Collinear vectors are those vectors that are parallel to the same line.

(iii) False.

Two vectors having the same magnitude need not necessarily be parallel to the same line. (iv) False.

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Only if the magnitude and direction of two vectors are the same, regardless of the positions of their initial points the two vector are said to be equal.

Exercise 10.2

1. Compute the magnitude of the following vectors:

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}; \quad \vec{b} = 2\hat{i} - 7\hat{j} - 3\hat{k}; \qquad \vec{c} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$$

Solution:

Given vectors are:

$$\begin{aligned} \vec{a} &= \hat{i} + \hat{j} + \hat{k}; \quad \vec{b} = 2\hat{i} - 7\hat{j} - 3\hat{k}; \qquad \vec{c} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}} \\ |\vec{a}| &= \sqrt{(1)^2 + (1)^2 + (1)^2} = \sqrt{3} \\ |\vec{b}| &= \sqrt{(2)^2 + (-7)^2 + (-3)^2} \\ &= \sqrt{4 + 49 + 9} \\ &= \sqrt{62} \\ |\vec{c}| &= \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(-\frac{1}{\sqrt{3}}\right)^2} \\ &= \sqrt{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} = 1 \end{aligned}$$

2. Write two different vectors having same magnitude. Solution:

Consider $\vec{a} = (\hat{i} - 2\hat{j} + 4\hat{k})$ and $\vec{b} = (2\hat{i} + \hat{j} - 4\hat{k})$. It can be observed that $|\vec{a}| = \sqrt{1^2 + (-2)^2 + 4^2} = \sqrt{1 + 4 + 16} = \sqrt{21}$ and $|\vec{b}| = \sqrt{2^2 + 1^2 + (-4)^2} = \sqrt{4 + 1 + 16} = \sqrt{21}$

Thus, \vec{a} and \vec{b} are two different vectors having the same magnitude. Here, the vectors are different as they have different directions.

3. Write two different vectors having same direction. Solution:

Consider
$$\vec{p} = (\hat{i} + \hat{j} + \hat{k})$$
 and $\vec{q} = (2\hat{i} + 2\hat{j} + 2\hat{k})$.

The direction cosines of \vec{p} are given by,

 $l = \frac{1}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{1}{\sqrt{3}}, \ m = \frac{1}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{1}{\sqrt{3}}, \ \text{and} \ n = \frac{1}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{1}{\sqrt{3}}$

The direction cosines of \vec{q} are given by

$$l = \frac{2}{\sqrt{2^2 + 2^2 + 2^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}, \ m = \frac{2}{\sqrt{2^2 + 2^2 + 2^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}},$$

and $n = \frac{2}{\sqrt{2^2 + 2^2 + 2^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}.$

4. Find the values of x and y so that the vectors $2\hat{i}+3\hat{j}$ and $x\hat{i}+y\hat{j}$ are equal Solution:

Given vectors $2\hat{i} + 3\hat{j}$ and $x\hat{i} + y\hat{j}$ will be equal only if their corresponding components are equal. Thus, the required values of x and y are 2 and 3 respectively.

5. Find the scalar and vector components of the vector with initial point (2, 1) and terminal point

(-5, 7). Solution:

The vector with initial point P (2, 1) and terminal point Q (-5, 7) can be shown as, $\overrightarrow{PQ} = (-5-2)\hat{i} + (7-1)\hat{j}$ $\overrightarrow{PQ} = -7\hat{i} + 6\hat{j}$

Thus, the required scalar components are -7 and 6 while the vector components are $-7\hat{i}$ and $6\hat{j}$.

6. Find the sum of the vectors $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$ Solution:

The given vectors are $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$ Hence, $\vec{a} + \vec{b} + \vec{c} = (1 - 2 + 1)\hat{i} + (-2 + 4 - 6)\hat{j} + (1 + 5 - 7)\hat{k}$

$$= 0 \cdot \hat{i} - 4\hat{j} - 1 \cdot \hat{k}$$
$$= -4\hat{j} - \hat{k}$$

7. Find the unit vector in the direction of the vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ Solution:

The unit vector \hat{a} in the direction of vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ is given by $\hat{a} = \frac{a}{|a|}$ So,

$$|\vec{a}| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{1 + 1 + 4} = \sqrt{6}$$

Thus,

 $\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{6}} = \frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k}$

8. Find the unit vector in the direction of vector \overrightarrow{PQ} , where P and Q are the points (1, 2, 3) and (4, 5, 6), respectively Solution:

Given points are P (1, 2, 3) and Q (4, 5, 6).
So,
$$\overrightarrow{PQ} = (4-1)\hat{i} + (5-2)\hat{j} + (6-3)\hat{k} = 3\hat{i} + 3\hat{j} + 3\hat{j} + 3\hat{j} + 3\hat{j} = \sqrt{9}\hat{j} + (6-3)\hat{k} = 3\hat{i} + 3\hat{j} + 3\hat{j} + 3\hat{j} + 3\hat{j} = \sqrt{9}\hat{j} = \sqrt{27} = 3\sqrt{3}$$

Thus, the unit vector in the direction of \overrightarrow{PQ} is
 $\overrightarrow{PQ} = \frac{3\hat{i} + 3\hat{j} + 3\hat{k}}{5} = \frac{1}{5}\hat{i} + \frac{1}{5}\hat{j} + \frac{1}{5}\hat{k}$

$$\frac{1}{|\overline{PQ}|} = \frac{1}{3\sqrt{3}} = \frac{1}{\sqrt{3}}i + \frac{1}{\sqrt{3}}j + \frac{1}{\sqrt{3}}k$$

9. For given vectors, $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$, find the unit vector in the direction of the vector $\vec{a} + \vec{b}$

Given vectors are
$$\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$$
 and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}^{-1}$
 $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$
 $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$
 $\therefore \vec{a} + \vec{b} = (2-1)\hat{i} + (-1+1)\hat{j} + (2-1)\hat{k} = 1\hat{i} + 0\hat{j} + 1\hat{k} = \hat{i} + \hat{k}$
 $|\vec{a} + \vec{b}| = \sqrt{1^2 + 1^2} = \sqrt{2}$
Thus, the unit vector in the direction of $(\vec{a} + \vec{b})$ is

Thus, the unit vector in the direction of $(\vec{a} + \vec{b})$ is $(\rightarrow \rightarrow)$

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ight|}=rac{\hat{i}+\hat{k}}{\sqrt{2}}=rac{1}{\sqrt{2}}\widehat{i}+rac{1}{\sqrt{2}}\widehat{k}.$$

10. Find a vector in the direction of vector $5\hat{i} - \hat{j} + 2\hat{k}$ which has magnitude 8 units. Solution:

3

Let
$$\vec{a} = 5\hat{i} - \hat{j} + 2\hat{k}$$
.
So,
 $|\vec{a}| = \sqrt{5^2 + (-1)^2 + 2^2} = \sqrt{25 + 1 + 4} = \sqrt{30}$
 $\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{5\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{30}}$

Thus, the vector in the direction of vector $5\hat{i} - \hat{j} + 2\hat{k}$ which has magnitude 8 units is given by,

$$8\hat{a} = 8\left(\frac{5\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{30}}\right) = \frac{40}{\sqrt{30}}\hat{i} - \frac{8}{\sqrt{30}}\hat{j} + \frac{16}{\sqrt{30}}\hat{k}$$
$$= 8\left(\frac{5\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{30}}\right)$$
$$= \frac{40}{\sqrt{30}}\hat{i} - \frac{8}{\sqrt{30}}\hat{j} + \frac{16}{\sqrt{30}}\hat{k}$$

11. Show that the vectors $2\hat{i}-3\hat{j}+4\hat{k}$ and $-4\hat{i}+6\hat{j}-8\hat{k}$ are collinear. Solution:

Let
$$\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$$
 and $\vec{b} = -4\hat{i} + 6\hat{j} - 8\hat{k}$.
It is seen that $\vec{b} = -4\hat{i} + 6\hat{j} - 8\hat{k} = -2(2\hat{i} - 3\hat{j} + 4\hat{k}) = -2\vec{a}$
 $\therefore \vec{b} = \lambda \vec{a}$
where,
 $\lambda = -2$

Therefore, the given vectors are collinear.

12. Find the direction cosines of the vector $\hat{i} + 2\hat{j} + 3\hat{k}$ Solution:

Let
$$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$$
.
The modulus is given by,
 $|\vec{a}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{1 + 4 + 9} = \sqrt{14}$
Thus, the direction cosines of \vec{a} are $\left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$.

13. Find the direction cosines of the vector joining the points A (1, 2, -3) and B (-1, -2, 1) directed from A to B.

Solution:

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Given points are A (1, 2, -3) and B (-1, -2, 1). Now, $\overrightarrow{AB} = (-1-1)\hat{i} + (-2-2)\hat{j} + \{1-(-3)\}\hat{k}$ $\overrightarrow{AB} = -2\hat{i} - 4\hat{j} + 4\hat{k}$ $|\overrightarrow{AB}| = \sqrt{(-2)^2 + (-4)^2 + 4^2} = \sqrt{4+16+16} = \sqrt{36} = 6$ Therefore, the direction cosines of \overrightarrow{AB} are $\left(-\frac{2}{6}, -\frac{4}{6}, \frac{4}{6}\right) = \left(-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}\right)$.

14. Show that the vector $\hat{i} + \hat{j} + \hat{k}$ is equally inclined to the axes OX, OY, and OZ. Solution:

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$.

Then,

$$\left|\vec{a}\right| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

Hence, the direction cosines of \vec{a} are $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$.

Now, let α , β , and γ be the angles formed by \vec{a} with the positive directions of x, y, and z axes.

So, we have
$$\cos \alpha = \frac{1}{\sqrt{3}}, \cos \beta = \frac{1}{\sqrt{3}}, \cos \gamma = \frac{1}{\sqrt{3}}.$$

Therefore, the given vector is equally inclined to axes OX, OY, and OZ.

15. Find the position vector of a point R which divides the line joining two points P and Q whose position

vectors are $\hat{i}+2\hat{j}-\hat{k}$ and $-\hat{i}+\hat{j}+\hat{k}$ respectively, in the ration 2:1 (i) internally (ii) externally Solution:

The position vector of point R dividing the line segment joining two points P and Q in the ratio *m*: *n* is given by:

(i) Internally: $\underline{m\vec{b} + n\vec{a}}$ $\underline{m+n}$

(ii) Externally: $\frac{m\vec{b} - n\vec{a}}{m - n}$

$$\overrightarrow{\text{OP}} = \hat{i} + 2\hat{j} - \hat{k}$$
 and $\overrightarrow{\text{OQ}} = -\hat{i} + \hat{j} + \hat{k}$

(i) The position vector of point R which divides the line joining two points P and Q internally in the ratio 2:1 is given by,

$$\overline{OR} = \frac{2(-\hat{i}+\hat{j}+\hat{k})+1(\hat{i}+2\hat{j}-\hat{k})}{2+1} = \frac{(-2\hat{i}+2\hat{j}+2\hat{k})+(\hat{i}+2\hat{j}-\hat{k})}{3}$$
$$= \frac{-\hat{i}+4\hat{j}+\hat{k}}{3} = -\frac{1}{3}\hat{i}+\frac{4}{3}\hat{j}+\frac{1}{3}\hat{k}$$

(ii) The position vector of point R which divides the line joining two points P and Q externally in the ratio 2:1 is given by, $2(\hat{i} + \hat{j} + \hat{i}) = 1(\hat{i} + 2\hat{j} + \hat{i})$

$$\overline{OR} = \frac{2(-i+j+k)-1(i+2j-k)}{2-1} = (-2\hat{i}+2\hat{j}+2\hat{k}) - (\hat{i}+2\hat{j}-\hat{k})$$
$$= -3\hat{i}+3\hat{k}$$

16. Find the position vector of the mid point of the vector joining the points P (2, 3, 4) and Q (4, 1, -2). Solution:

The position vector of mid-point R of the vector joining points P (2, 3, 4) and Q (4, 1, -2) is given by,

$$\overline{OR} = \frac{\left(2\hat{i}+3\hat{j}+4\hat{k}\right) + \left(4\hat{i}+\hat{j}-2\hat{k}\right)}{2} = \frac{\left(2+4\right)\hat{i}+\left(3+1\right)\hat{j}+\left(4-2\right)\hat{k}}{2} = \frac{6\hat{i}+4\hat{j}+2\hat{k}}{2} = 3\hat{i}+2\hat{j}+\hat{k}$$

17. Show that the points A, B and C with position vectors, $\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$, respectively form the vertices of a right angled triangle. Solution:

Given position vectors of points A, B, and C are:

 $\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}, \ \vec{b} = 2\hat{i} - \hat{j} + \hat{k} \text{ and } \vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$ $\therefore \overrightarrow{AB} = \vec{b} - \vec{a} = (2-3)\hat{i} + (-1+4)\hat{j} + (1+4)\hat{k} = -\hat{i} + 3\hat{j} + 5\hat{k}$ $\overrightarrow{BC} = \vec{c} - \vec{b} = (1-2)\hat{i} + (-3+1)\hat{j} + (-5-1)\hat{k} = -\hat{i} - 2\hat{j} - 6\hat{k}$ $\overrightarrow{CA} = \vec{a} - \vec{c} = (3-1)\hat{i} + (-4+3)\hat{j} + (-4+5)\hat{k} = 2\hat{i} - \hat{j} + \hat{k}$ Now, $\left|\overrightarrow{AB}\right|^2 = (-1)^2 + 3^2 + 5^2 = 1 + 9 + 25 = 35$ $\left|\overrightarrow{BC}\right|^2 = (-1)^2 + (-2)^2 + (-6)^2 = 1 + 4 + 36 = 41$ $\left|\overrightarrow{CA}\right|^2 = 2^2 + (-1)^2 + 1^2 = 4 + 1 + 1 = 6$ Hence, $\left|\overrightarrow{AB}\right|^2 + \left|\overrightarrow{CA}\right|^2 = 35 + 6 = 41 = \left|\overrightarrow{BC}\right|^2$

18. In triangle ABC (Fig 10.18) which of the following is not true:

- (A) $\overline{AB} + \overline{BC} + \overline{CA} = \vec{0}$ (B) $\overline{AB} + \overline{BC} - \overline{AC} = \vec{0}$ (C) $\overline{AB} + \overline{BC} - \overline{AC} = \vec{0}$
- (D) $\overrightarrow{AB} \overrightarrow{CB} + \overrightarrow{CA} = \overrightarrow{0}$ Solution:

C

Fig 10.18

Applying the triangle law of addition in the given triangle, we get: AB + BC = AC...(1) $\overrightarrow{AB} + \overrightarrow{BC} = -\overrightarrow{CA}$ $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{0}$...(2) ... The equation given in alternative A is true. AB + BC = AC $\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \overrightarrow{0}$... The equation given in alternative B is true. From equation (2), we have: $\overrightarrow{AB} - \overrightarrow{CB} + \overrightarrow{CA} = \overrightarrow{0}$... The equation given in alternative D is true. Now, consider the equation given in alternative C: $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{CA} = \overrightarrow{0}$ $\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{CA}$...(3) From equations (1) and (3), we get: AC = CA $\overrightarrow{AC} = -\overrightarrow{AC}$ $\overrightarrow{AC} + \overrightarrow{AC} = \overrightarrow{0}$ $2\overrightarrow{AC} = \overrightarrow{0}$ $\overrightarrow{AC} = \overrightarrow{0}$, which is not true. Thus, the equation given in alternative C is incorrect. The correct answer is C. 19. If \vec{a} and \vec{b} are two collinear vectors, then which of the following are A. $\vec{b} = \lambda \vec{a}$ incorrect: $\vec{a} = \pm \vec{b}$, for some scalar λ **B**. C. the respective components of \vec{a} and proportional D. both the vectors \vec{a} have same direction, but different magnitudes Solution: If \vec{a} and \vec{b} are two collinear vectors, then they are parallel. So, we have:

 $\vec{b} = \lambda \vec{a}$ (For some scalar λ) If $\lambda = \pm 1$, then $\vec{a} = \pm \vec{b}$. If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then
$$\begin{split} \vec{b} &= \lambda \vec{a}, \\ b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k} &= \lambda \left(a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k} \right) \\ b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k} &= (\lambda a_1) \hat{i} + (\lambda a_2) \hat{j} + (\lambda a_3) \hat{k} \\ b_1 &= \lambda a_1, b_2 = \lambda a_2, b_3 = \lambda a_3 \\ \frac{b_1}{a_1} &= \frac{b_2}{a_2} = \frac{b_3}{a_3} = \lambda \end{split}$$

Hence, the respective components of \vec{a} and \vec{b} are proportional.

But, vectors \vec{a} and \vec{b} can have different directions.

Thus, the statement given in D is incorrect.

The correct answer is D.

Exercise 10.3

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1. Find the angle between two vectors \vec{a} and \vec{b} with magnitudes $\sqrt{3}$ and 2, respectively having $\vec{a} \cdot \vec{b} = \sqrt{6}$.

Solution:

$$|\vec{a}| = \sqrt{3}, |\vec{b}| = 2 \text{ and}, \vec{a} \cdot \vec{b} = \sqrt{6}$$

Now, we know that $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$.
 $\therefore \sqrt{6} = \sqrt{3} \times 2 \times \cos \theta$
 $\cos \theta = \frac{\sqrt{6}}{\sqrt{3} \times 2}$
 $\cos \theta = \frac{1}{\sqrt{2}}$
 $\Rightarrow \theta = \frac{\pi}{4}$

Thus , the angle between the given vectors \vec{a} and \vec{b} is $\frac{\pi}{4}$

2. Find the angle between the vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$ Solution:

Given vectors are:
$$\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$$
 and $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$
 $|\vec{a}| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{1 + 4 + 9} = \sqrt{14}$
 $|\vec{b}| = \sqrt{3^2 + (-2)^2 + 1^2} = \sqrt{9 + 4 + 1} = \sqrt{14}$
Now, $\vec{a} \cdot \vec{b} = (\hat{i} - 2\hat{j} + 3\hat{k})(3\hat{i} - 2\hat{j} + \hat{k})$
 $= 1.3 + (-2)(-2) + 3.1$
 $= 3 + 4 + 3$
 $= 10$
Also, we know that $\vec{a} \cdot b = |\vec{a}| |b| \cos \theta$.
 $\therefore 10 = \sqrt{14}\sqrt{14} \cos \theta$
 $\cos \theta = \frac{10}{14}$
 $\theta = \cos^{-1}(\frac{5}{-1})$

3. Find the projection of the vector $\hat{i} - \hat{j}$ on the vector $\hat{i} + \hat{j}$ Solution:

Let $\vec{a} = \hat{i} - \hat{j}$ and $\vec{b} = \hat{i} + \hat{j}$. Now, projection of vector \vec{a} on \vec{b} is given by, $\frac{1}{|\vec{b}|} (\vec{a}.\vec{b}) = \frac{1}{\sqrt{1+1}} \{1.1 + (-1)(1)\} = \frac{1}{\sqrt{2}} (1-1) = 0$

(7)

Thus, the projection of vector \vec{a} on \vec{b} is 0.

4. Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $7\hat{i} - \hat{j} + 8\hat{k}$. Solution:

Let
$$\vec{a} = \hat{i} + 3\hat{j} + 7\hat{k}$$
 and $\hat{b} = 7\hat{i} - \hat{j} + 8\hat{k}$.

Now, projection of vector \vec{a} on \vec{b} is given by,

$$\frac{1}{\left|\vec{b}\right|}\left(\vec{a}\cdot\vec{b}\right) = \frac{1}{\sqrt{7^2 + (-1)^2 + 8^2}}\left\{1(7) + 3(-1) + 7(8)\right\} = \frac{7 - 3 + 56}{\sqrt{49 + 1 + 64}} = \frac{60}{\sqrt{114}}$$

5. Show that each of the given three vectors is a unit vector:

$$\frac{1}{7} \Big(2\hat{i} + 3\hat{j} + 6\hat{k} \Big), \frac{1}{7} \Big(3\hat{i} - 6\hat{j} + 2\hat{k} \Big), \frac{1}{7} \Big(6\hat{i} + 2\hat{j} - 3\hat{k} \Big)$$

Also, show that they are mutually perpendicular to each other. Solution:

Let
$$\vec{a} = \frac{1}{7} \left(2\hat{i} + 3\hat{j} + 6\hat{k} \right) = \frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k},$$

 $\vec{b} = \frac{1}{7} \left(3\hat{i} - 6\hat{j} + 2\hat{k} \right) = \frac{3}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{2}{7}\hat{k},$
 $\vec{c} = \frac{1}{7} \left(6\hat{i} + 2\hat{j} - 3\hat{k} \right) = \frac{6}{7}\hat{i} + \frac{2}{7}\hat{j} - \frac{3}{7}\hat{k}.$
 $|\vec{a}| = \sqrt{\left(\frac{2}{7}\right)^2 + \left(\frac{3}{7}\right)^2 + \left(\frac{6}{7}\right)^2} = \sqrt{\frac{4}{49} + \frac{9}{49} + \frac{36}{49}} = 1$
 $|\vec{b}| = \sqrt{\left(\frac{3}{7}\right)^2 + \left(-\frac{6}{7}\right)^2 + \left(\frac{2}{7}\right)^2} = \sqrt{\frac{9}{49} + \frac{36}{49} + \frac{4}{49}} = 1$
 $|\vec{c}| = \sqrt{\left(\frac{6}{7}\right)^2 + \left(\frac{2}{7}\right)^2 + \left(-\frac{3}{7}\right)^2} = \sqrt{\frac{36}{49} + \frac{4}{49} + \frac{9}{49}} = 1$

Hence, each of the given three vectors is a unit vector.

$$\vec{a} \cdot \vec{b} = \frac{2}{7} \times \frac{3}{7} + \frac{3}{7} \times \left(\frac{-6}{7}\right) + \frac{6}{7} \times \frac{2}{7} = \frac{6}{49} - \frac{18}{49} + \frac{12}{49} = 0$$
$$\vec{b} \cdot \vec{c} = \frac{3}{7} \times \frac{6}{7} + \left(\frac{-6}{7}\right) \times \frac{2}{7} + \frac{2}{7} \times \left(\frac{-3}{7}\right) = \frac{18}{49} - \frac{12}{49} - \frac{6}{49} = 0$$
$$\vec{c} \cdot \vec{a} = \frac{6}{7} \times \frac{2}{7} + \frac{2}{7} \times \frac{3}{7} + \left(\frac{-3}{7}\right) \times \frac{6}{7} = \frac{12}{49} + \frac{6}{49} - \frac{18}{49} = 0$$

Therefore, the given three vectors are mutually perpendicular to each other.

4

$$|\vec{a}|$$
 and $|\vec{b}|$, if $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})=8$ and $|\vec{a}|=8|\vec{b}|$ 6. Find

 $\begin{aligned} \left(\vec{a} \cdot \vec{b}\right) \cdot \left(\vec{a} - \vec{b}\right) &= 8\\ \vec{a} \cdot \vec{a} - \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} - \vec{b} \cdot \vec{b} &= 8\\ \left|\vec{a}\right|^2 - \left|\vec{b}\right|^2 &= 8\\ \left(8\left|\vec{b}\right|\right)^2 - \left|\vec{b}\right|^2 &= 8\\ \left(8\left|\vec{b}\right|\right)^2 - \left|\vec{b}\right|^2 &= 8\\ 63\left|\vec{b}\right|^2 - \left|\vec{b}\right|^2 &= 8\\ 63\left|\vec{b}\right|^2 &= 8\\ \left|\vec{b}\right|^2 &= \frac{8}{63}\\ \left|\vec{b}\right| &= \sqrt{\frac{8}{63}} \qquad [Magnitude of a vector is non-negative]\\ \left|\vec{b}\right| &= \frac{2\sqrt{2}}{3\sqrt{7}}\\ And,\\ \left|\vec{a}\right| &= 8\left|\vec{b}\right| &= \frac{8 \times 2\sqrt{2}}{3\sqrt{7}} = \frac{16\sqrt{2}}{3\sqrt{7}} \end{aligned}$

7. Evaluate the product $(3\vec{a}-5\vec{b})\cdot(2\vec{a}+7\vec{b})$ Solution:

 $(3\vec{a} - 5\vec{b}) \cdot (2\vec{a} + 7\vec{b})$ $= 3\vec{a} \cdot 2\vec{a} + 3\vec{a} \cdot 7\vec{b} - 5\vec{b} \cdot 2\vec{a} - 5\vec{b} \cdot 7\vec{b}$ $= 6\vec{a} \cdot \vec{a} + 21\vec{a} \cdot \vec{b} - 10\vec{a} \cdot \vec{b} - 35\vec{b} \cdot \vec{b}$ $= 6|\vec{a}|^2 + 11\vec{a} \cdot \vec{b} - 35|\vec{b}|^2$

8. Find the magnitude of two vectors \vec{a} and \vec{b} , having the same magnitude and such that the angle between them is 60° and their scalar product is $\frac{1}{2}$.

Let θ be the angle between the vectors \vec{a} and \vec{b} . It is given that $|\vec{a}| = |\vec{b}|$, $\vec{a} \cdot \vec{b} = \frac{1}{2}$, and $\theta = 60^{\circ}$(1) We know that $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$. $\therefore \frac{1}{2} = |\vec{a}| |\vec{a}| \cos 60^{\circ}$ [Using (1)] $\frac{1}{2} = |\vec{a}|^2 \times \frac{1}{2}$ $|\vec{a}|^2 = 1$ $|\vec{a}| = |\vec{b}| = 1$

9. Find $|\vec{x}|$, if for a unit vector \vec{a} , $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 12$ Solution:

 $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 12$ $\vec{x} \cdot \vec{x} + \vec{x} \cdot \vec{a} - \vec{a} \cdot \vec{x} - \vec{a} \cdot \vec{a} = 12$ $|\vec{x}|^2 - |\vec{a}|^2 = 12$ $|\vec{x}|^2 - 1 = 12$ $|\vec{x}|^2 = 13$ $\therefore |\vec{x}| = \sqrt{13}$

10. If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda \vec{b}$ is perpendicular to \vec{c} , then find the value of λ .

Given vectors are $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$, and $\vec{c} = 3\hat{i} + \hat{j}$. Now, $\vec{a} + \lambda \vec{b} = (2\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda (-\hat{i} + 2\hat{j} + \hat{k}) = (2 - \lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (3 + \lambda)\hat{k}$ If $(\vec{a} + \lambda \vec{b})$ is perpendicular to \vec{c} , then $(\vec{a} + \lambda \vec{b}) \cdot \vec{c} = 0$. $[(2 - \lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (3 + \lambda)\hat{k}] \cdot (3\hat{i} + \hat{j}) = 0$ $(2 - \lambda)3 + (2 + 2\lambda)1 + (3 + \lambda)0 = 0$ $6 - 3\lambda + 2 + 2\lambda = 0$ $-\lambda + 8 = 0$ $\lambda = 8$ Therefore, the required value of λ is 8.

11. Show that $\frac{|\vec{a}|\vec{b}+|\vec{b}|\vec{a}}{|\vec{a}|\vec{b}-|\vec{b}|\vec{a}}$, for any two nonzero vectors \vec{a} and \vec{b} . Solution:

 $\begin{aligned} \left(\left| \vec{a} \right| \vec{b} + \left| \vec{b} \right| \vec{a} \right) \cdot \left(\left| \vec{a} \right| \vec{b} - \left| \vec{b} \right| \vec{a} \right) \\ &= \left| \vec{a} \right|^2 \vec{b} \cdot \vec{b} - \left| \vec{a} \right| \left| \vec{b} \right| \vec{b} \cdot \vec{a} + \left| \vec{b} \right| \left| \vec{a} \right| \vec{a} \cdot \vec{b} - \left| \vec{b} \right|^2 \vec{a} \cdot \vec{a} \\ &= \left| \vec{a} \right|^2 \left| \vec{b} \right|^2 - \left| \vec{b} \right|^2 \left| \vec{a} \right|^2 \\ &= 0 \end{aligned}$

Therefore, $|\vec{a}|\vec{b} + |\vec{b}|\vec{a}$ and $|\vec{a}|\vec{b} - |\vec{b}|\vec{a}$ are perpendicular to each other.

12. If $\vec{a} \cdot \vec{a} = 0$ and $\vec{a} \cdot \vec{b} = 0$, then what can be concluded about the vector \vec{b} ? Solution:

Given, $\vec{a} \cdot \vec{a} = 0$ and $\vec{a} \cdot \vec{b} = 0$. Now, $\vec{a} \cdot \vec{a} = 0 \Rightarrow |\vec{a}|^2 = 0 \Rightarrow |\vec{a}| = 0$

 \vec{a} is a zero vector. Thus, vector \vec{b} satisfying $\vec{a} \cdot \vec{b} = 0$ can be any vector.

13. If \vec{a} , \vec{b} and \vec{c} are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$. Solution:

Given, $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. So, $\vec{a} + \vec{c} + \vec{c} = \vec{0}$.				
$\vec{a} \cdot \vec{a} + \vec{b} + \vec{c} = \vec{a} \cdot \vec{0}$ $\vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} = \vec{a} \cdot \vec{0}$	[[Distributivity of so	calar product over addition]	
$1 + \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} = 0$	(1)	$\vec{a} \cdot \vec{a} = \vec{a} \cdot \vec{a} \cos 0^\circ$ \vec{a} is unit vector =	$\dot{a} = 1$ $\Rightarrow \vec{a} = 1$	
Next, $\vec{b} \cdot (\vec{a} + \vec{b} + \vec{c}) = \vec{b} \cdot \vec{0}$	L			
$\vec{b}\cdot\vec{a}+\vec{b}\cdot\vec{b}+\vec{b}\cdot\vec{c}=\vec{b}\cdot\vec{0}$				
$\vec{b} \cdot \vec{a} + 1 + \vec{b} \cdot \vec{c} = 0$ And, $\vec{c} \cdot \left(\vec{a} + \vec{b} + \vec{c}\right) = \vec{c} \cdot \vec{0}$ $\vec{c} \cdot \vec{a} + \vec{c} \cdot \vec{b} + \vec{c} \cdot \vec{c} = \vec{c} \cdot \vec{0}$	(2)	$\left[\vec{b}\cdot\vec{b}=1\right]$		
$\vec{c} \cdot \vec{a} + \vec{c} \cdot \vec{b} + 1 = 0$ From (1), (2) and (3),	(3)	$\begin{bmatrix} \vec{c} \cdot \vec{c} = 1 \end{bmatrix}$		
$(1+\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c})+(\vec{b}\cdot\vec{a}+\vec{c})$	$(1+b\cdot \tilde{c})+(\tilde{c})$	$\cdot \vec{a} + \vec{c} \cdot \vec{b} + 1 = 0 + $	- 0 + 0	
$(3+\vec{a}\cdot\vec{b}+\vec{c}\cdot\vec{a})+(\vec{a}\cdot\vec{b}+\vec{c}\cdot\vec{a})$	$(\vec{b}\cdot\vec{c})+(\vec{c}\cdot\vec{a})$	$+\vec{b}\cdot\vec{c}=0$	[Scalar product is commutative	•]
$3+2\left(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}\right)$	= 0			
$\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=\frac{-3}{2}$				

14. If either vector $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$, then $\vec{a} \cdot \vec{b} = 0$. But the converse need not be true. Justify your answer with an example. Solution:

Consider $\vec{a} = 2\hat{i} + 4\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 3\hat{j} - 6\hat{k}$. Then, their dot product is given by: $\vec{a} \cdot \vec{b} = 2.3 + 4.3 + 3(-6) = 6 + 12 - 18 = 0$ Now, it's seen that $|\vec{a}| = \sqrt{2^2 + 4^2 + 3^2} = \sqrt{29}$ $\therefore \vec{a} \neq \vec{0}$ $|\vec{b}| = \sqrt{3^2 + 3^2 + (-6)^2} = \sqrt{54}$ $\therefore \vec{b} \neq \vec{0}$ Therefore, the approximate for a final statement product of the st

Therefore, the converse of the given statement need not be true.

15. If the vertices A, B, C of a triangle ABC are (1, 2, 3), (-1, 0, 0), (0, 1, 2), respectively, then find

 $\angle ABC$. [$\angle ABC$ is the angle between the vectors \overline{BA} and \overline{BC}] Solution:

The vertices of $\triangle ABC$ are given as A (1, 2, 3), B (-1, 0, 0), and C (0, 1, 2). Also given, $\angle ABC$ is the angle between the vectors \overline{BA} and \overline{BC} . $\overline{BA} = \{1 - (-1)\}\hat{i} + (2 - 0)\hat{j} + (3 - 0)\hat{k} = 2\hat{i} + 2\hat{j} + 3\hat{k}$ $\overline{BC} = \{0 - (-1)\}\hat{i} + (1 - 0)\hat{j} + (2 - 0)\hat{k} = \hat{i} + \hat{j} + 2\hat{k}$ $\therefore \overline{BA} \cdot \overline{BC} = (2\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (\hat{i} + \hat{j} + 2\hat{k}) = 2 \times 1 + 2 \times 1 + 3 \times 2 = 2 + 2 + 6 = 10$ $|\overline{BA}| = \sqrt{2^2 + 2^2 + 3^2} = \sqrt{4 + 4 + 9} = \sqrt{17}$ $|\overline{BC}| = \sqrt{1 + 1 + 2^2} = \sqrt{6}$ Now, we know that $\overline{BA} \cdot \overline{BC} = |\overline{BA}| |\overline{BC}| \cos(\angle ABC)$. $\therefore 10 = \sqrt{17} \times \sqrt{6} \cos(\angle ABC)$ $\cos(\angle ABC) = \frac{10}{\sqrt{17} \times \sqrt{6}}$ $\angle ABC = \cos^{-1}\left(\frac{10}{\sqrt{102}}\right)$

16. Show that the points A (1, 2, 7), B (2, 6, 3) and C (3, 10, -1) are collinear. Solution:

Given points are A (1, 2, 7), B (2, 6, 3), and C (3, 10, -1). Now,

$$\overline{AB} = (2-1)\hat{i} + (6-2)\hat{j} + (3-7)\hat{k} = \hat{i} + 4\hat{j} - 4\hat{k}$$

$$\overline{BC} = (3-2)\hat{i} + (10-6)\hat{j} + (-1-3)\hat{k} = \hat{i} + 4\hat{j} - 4\hat{k}$$

$$\overline{AC} = (3-1)\hat{i} + (10-2)\hat{j} + (-1-7)\hat{k} = 2\hat{i} + 8\hat{j} - 8\hat{k}$$

Now,

$$\left|\overline{AB}\right| = \sqrt{1^2 + 4^2 + (-4)^2} = \sqrt{1 + 16 + 16} = \sqrt{33}$$

$$\left|\overline{BC}\right| = \sqrt{1^2 + 4^2 + (-4)^2} = \sqrt{1 + 16 + 16} = \sqrt{33}$$

$$\left|\overline{AC}\right| = \sqrt{2^2 + 8^2 + 8^2} = \sqrt{4 + 64 + 64} = \sqrt{132} = 2\sqrt{33}$$

$$\therefore \left|\overline{AC}\right| = \left|\overline{AB}\right| + \left|\overline{BC}\right|$$

Therefore, the given points A, B, and C are collinear.

17. Show that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ form the vertices of a right angled triangle. Solution:

Let vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ be position vectors of points A, B, and C respectively. So, $\overrightarrow{OA} = 2\hat{i} - \hat{j} + \hat{k}$, $\overrightarrow{OB} = \hat{i} - 3\hat{j} - 5\hat{k}$ and $\overrightarrow{OC} = 3\hat{i} - 4\hat{j} - 4\hat{k}$ Now, vectors \overrightarrow{AB} , \overrightarrow{BC} , and \overrightarrow{AC} represent the sides of $\triangle ABC$. Hence, $\overrightarrow{AB} = (1-2)\hat{i} + (-3+1)\hat{j} + (-5-1)\hat{k} = -\hat{i} - 2\hat{j} - 6\hat{k}$ $\overrightarrow{BC} = (3-1)\hat{i} + (-4+3)\hat{j} + (-4+5)\hat{k} = 2\hat{i} - \hat{j} + \hat{k}$ $\overrightarrow{AC} = (2-3)\hat{i} + (-1+4)\hat{j} + (1+4)\hat{k} = -\hat{i} + 3\hat{j} + 5\hat{k}$ $\left|\overrightarrow{AB}\right| = \sqrt{(-1)^2 + (-2)^2 + (-6)^2} = \sqrt{1+4+36} = \sqrt{41}$ $\left|\overrightarrow{BC}\right| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{4+1+1} = \sqrt{6}$ $\left|\overrightarrow{AC}\right| = \sqrt{(-1)^2 + 3^2 + 5^2} = \sqrt{1+9+25} = \sqrt{35}$ $\therefore \left|\overrightarrow{BC}\right|^2 + \left|\overrightarrow{AC}\right|^2 = 6+35 = 41 = \left|\overrightarrow{AB}\right|^2$ Therefore, $\triangle ABC$ is a right-angled triangle.

18. If \vec{a} is a nonzero vector of magnitude 'a' and λ a nonzero scalar, then $\lambda^{\vec{a}}$ (A) $\lambda = 1$ (B) $\lambda = -1$ (C) $a = |\lambda|$ (D) $a = 1/|\lambda|$ is unit vector if Solution:



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Exercise 10.4

The correct answer is D.

1. Find $|\vec{a} \times \vec{b}|$, if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ Solution:

Therefore, vector $\lambda \vec{a}$ is a unit vector if $a = \frac{1}{|\lambda|}$

We have,

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$$\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k} \text{ and } \vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{vmatrix}$$

$$= \hat{i}(-14 + 14) - \hat{j}(2 - 21) + \hat{k}(-2 + 21) = 19\hat{j} + 19\hat{k}$$
Therefore,
$$|\vec{a} \times \vec{b}| = \sqrt{(19)^2 + (19)^2} = \sqrt{2 \times (19)^2} = 19\sqrt{2}$$

2. Find a unit vector perpendicular to each of the vector $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$, where $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$.

Solution:

We have,

$$\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k} \text{ and } \vec{b} = -\hat{i} + 2\hat{j} - 2\hat{k}$$

So, we have
$$\vec{a} + \vec{b} = 4\hat{i} + 4\hat{j}, \ \vec{a} - \vec{b} = 2\hat{i} + 4\hat{k}$$

$$\left(\vec{a} + \vec{b}\right) \times \left(\vec{a} - \vec{b}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix} = \hat{i}(16) - \hat{j}(16) + \hat{k}(-8) = 16\hat{i} - 16\hat{j} - 8\hat{k}$$

Thus,
$$\left| \left(\vec{a} + \vec{b}\right) \times \left(\vec{a} - \vec{b}\right) \right| = \sqrt{16^2 + (-16)^2 + (-8)^2}$$

$$= \sqrt{2^2 \times 8^2 + 2^2 \times 8^2 + 8^2}$$

$$8\sqrt{2^2+2^2+1}=8\sqrt{9}=8\times 3=24$$

 \vec{a}

Therefore, the unit vector perpendicular to each of the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ is given by the relation,

$$=\pm \frac{(\vec{a}+\vec{b}) \times (\vec{a}-\vec{b})}{\left|(\vec{a}+\vec{b}) \times (\vec{a}-\vec{b})\right|} =\pm \frac{16\hat{i}-16\hat{j}-8\hat{k}}{24}$$
$$=\pm \frac{2\hat{i}-2\hat{j}-\hat{k}}{3} =\pm \frac{2}{3}\hat{i} \mp \frac{2}{3}\hat{j} \mp \frac{1}{3}\hat{k}$$

=

3. If a unit vector \vec{a} makes an

find θ and hence, the compounds of Solution:

 $\frac{\pi}{3}$ with $\hat{i}, \frac{\pi}{4}$ with \hat{j} angles and an acute angle θ with \hat{k} , then

Let unit vector \vec{a} have (a_1, a_2, a_3) components.

$$\Rightarrow \vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$
As \vec{a} is a unit vector, $|\vec{a}| = 1$.
Also given, that \vec{a} makes angles $\frac{\pi}{3}$ with $\hat{i}, \frac{\pi}{4}$ with \hat{j} , and an acute angle θ with \hat{k} .
Then, we have
 $\cos \frac{\pi}{3} = \frac{a_1}{|\vec{a}|}$
 $\Rightarrow \frac{1}{2} = a_1$ $[|\vec{a}| = 1]$
 $\cos \frac{\pi}{4} = \frac{a_2}{|\vec{a}|}$
 $\Rightarrow \frac{1}{\sqrt{2}} = a_2$ $[|\vec{a}| = 1]$
Also, $\cos \theta = \frac{a_3}{|\vec{a}|}$.

$$\begin{aligned} \cos^{3} &= |\vec{a}| \\ \Rightarrow \frac{1}{2} = a_{1} \qquad [|\vec{a}| = 1] \\ \cos\frac{\pi}{4} &= \frac{a_{2}}{|\vec{a}|} \\ \Rightarrow \frac{1}{\sqrt{2}} = a_{2} \qquad [|\vec{a}| = 1] \\ \text{Also, } \cos\theta &= \frac{a_{3}}{|\vec{a}|} \\ \Rightarrow a_{3} &= \cos\theta \qquad [|\vec{a}|] \\ \text{Now.} \\ |\vec{a}| = 1 \\ \sqrt{a_{1}^{2} + a_{2}^{2} + a_{3}^{2}} = 1 \end{aligned}$$
$$\begin{aligned} \left(\frac{1}{2}\right)^{2} + \left(\frac{1}{\sqrt{2}}\right)^{2} + \cos^{2}\theta = 1 \\ \frac{1}{4} + \frac{1}{2} + \cos^{2}\theta = 1 \\ \frac{1}{4} + \frac{1}{2} + \cos^{2}\theta = 1 \\ \cos^{2}\theta = 1 - \frac{3}{4} = \frac{1}{4} \\ \cos\theta &= \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3} \\ \therefore a_{3} &= \cos\frac{\pi}{3} = \frac{1}{2} \end{aligned}$$
Thus, $\theta = \frac{\pi}{3}$ and the components of \vec{a} are $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}\right)$

4. Show that $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b})$

Taking the LHS, we have

$$(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})$$

= $(\vec{a} - \vec{b}) \times \vec{a} + (\vec{a} - \vec{b}) \times \vec{b}$
= $\vec{a} \times \vec{a} - \vec{b} \times \vec{a} + \vec{a} \times \vec{b} - \vec{b} \times \vec{b}$
= $\vec{0} + \vec{a} \times \vec{b} + \vec{a} \times \vec{b} - \vec{0}$
= $2(\vec{a} \times \vec{b})$

[By distributivity of vector product over addition] [Again, by distributivity of vector product over addition]

5. Find
$$\lambda$$
 and μ if $(2\hat{i}+6\hat{j}+27\hat{k})\times(\hat{i}+\lambda\hat{j}+\mu\hat{k})=\vec{0}$.
Solution:

 $\vec{a} \cdot \vec{b} = 0$ Then, (i) Either $|\vec{a}| = 0$ or $|\vec{b}| = 0$, or $\vec{a} \perp \vec{b}$ (in case \vec{a} and \vec{b} are non-zero) $\vec{a} \times \vec{b} = 0$ (ii) Either $|\vec{a}| = 0$ or $|\vec{b}| = 0$, or $\vec{a} \parallel \vec{b}$ (in case \vec{a} and \vec{b} are non-zero) But, \vec{a} and \vec{b} cannot be perpendicular and parallel simultaneously. Therefore, $|\vec{a}| = 0$ or $|\vec{b}| = 0$. 7. Let the vectors \vec{a} , \vec{b} , \vec{c} given as $a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, $c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ that $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$

Solution:

We have,

. Then show

Take any parallel non-zero vectors so that $\vec{a} \times \vec{b} = \vec{0}$. Let $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{b} = 4\hat{i} + 6\hat{j} + 8\hat{k}$. Then, $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 4 & 6 & 8 \end{vmatrix} = \hat{i} (24 - 24) - \hat{j} (16 - 16) + \hat{k} (12 - 12) = 0\hat{i} + 0\hat{j} + 0\hat{k} = \vec{0}$ Now, it's seen that

$$\begin{vmatrix} \vec{a} \end{vmatrix} = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}$$

$$\therefore \vec{a} \neq \vec{0}$$

$$\begin{vmatrix} \vec{b} \end{vmatrix} = \sqrt{4^2 + 6^2 + 8^2} = \sqrt{116}$$

$$\therefore \vec{b} \neq \vec{0}$$

Thus, the converse of the given statement need not be true.

9. Find the area of the triangle with vertices A (1, 1, 2), B (2, 3, 5) and C (1, 5, 5). Solution:

Given A (1, 1, 2), B (2, 3, 5) and C (1, 5, 5) are the vertices of triangle ABC. The adjacent sides \overrightarrow{AB} and \overrightarrow{BC} of $\triangle ABC$ are given as:

$$\overrightarrow{AB} = (2-1)\hat{i} + (3-1)\hat{j} + (5-2)\hat{k} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\overrightarrow{BC} = (1-2)\hat{i} + (5-3)\hat{j} + (5-5)\hat{k} = -\hat{i} + 2\hat{j}$$
Now,
Area of $\triangle ABC = \frac{1}{2} |\overrightarrow{AB} \times \overrightarrow{BC}|$

$$\overrightarrow{AB} \times \overrightarrow{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -1 & 2 & 0 \end{vmatrix} = \hat{i} (-6) - \hat{j} (3) + \hat{k} (2+2) = -6\hat{i} - 3\hat{j} + 4\hat{k}$$

$$\therefore |\overrightarrow{AB} \times \overrightarrow{BC}| = \sqrt{(-6)^2 + (-3)^2 + 4^2} = \sqrt{36 + 9 + 16} = \sqrt{61}$$
Therefore, the area of $\triangle ABC$ is $\frac{\sqrt{61}}{2}$ square units.

10. Find the area of the parallelogram whose adjacent sides are determined by the vector $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$. Solution: The area of the parallelogram whose adjacent sides are \vec{a} and \vec{b} is $|\vec{a} \times \vec{b}|$. Now, the adjacent sides are given as:

$$\vec{a} = \hat{i} - \hat{j} + 3\hat{k} \text{ and } \vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$$

$$\therefore \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix} = \hat{i} (-1 + 21) - \hat{j} (1 - 6) + \hat{k} (-7 + 2) = 20\hat{i} + 5\hat{j} - 5\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{20^2 + 5^2 + 5^2} = \sqrt{400 + 25 + 25} = 15\sqrt{2}$$

Therefore, the area of the given parallelogram is $15\sqrt{2}$ square units -

11. Let the vectors \vec{a} and \vec{b} be such that $|\vec{a}| = 3$ and $|\vec{b}| = \frac{\sqrt{2}}{3}$, then $\vec{a} \times \vec{b}$ between \vec{a} and \vec{b} is (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$ Solution:

is a unit vector, if the angle

Given, $\left|\vec{a}\right| = 3$ and $\left|\vec{b}\right| = \frac{\sqrt{2}}{3}$.

We know that $\vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \sin \theta \hat{n}$, where \hat{n} is a unit vector perpendicular to both \vec{a} and \vec{b} and θ is the angle between \vec{a} and \vec{b}

Now, $\vec{a} \times \vec{b}$ is a unit vector if $|\vec{a} \times \vec{b}| = 1$.

$$\begin{vmatrix} \vec{a} \times \vec{b} \end{vmatrix} = 1$$

$$\begin{vmatrix} \vec{a} \\ |\vec{b}| \sin \theta \hat{n} \end{vmatrix} = 1$$

$$\begin{vmatrix} \vec{a} \\ |\vec{b}| \sin \theta \end{vmatrix} = 1$$

$$3 \times \frac{\sqrt{2}}{3} \times \sin \theta = 1$$

$$\sin \theta = \frac{1}{\sqrt{2}}$$

$$\theta = \frac{\pi}{4}$$

Thus, $\vec{a} \times \vec{b}$ is a unit vector if the angle between \vec{a} and \vec{b} is $\frac{\pi}{4}$.

So, the correct answer is B.

12. Area of a rectangle having vertices A, B, C, and D with position

vectors

(C) 2

(A) $\frac{1}{2}$

respectively is

Solution:

The position vectors of vertices A, B, C, and D of rectangle ABCD are given as: $\overrightarrow{\text{OA}} = -\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}, \ \overrightarrow{\text{OB}} = \hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}, \ \overrightarrow{\text{OC}} = \hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}, \ \overrightarrow{\text{OD}} = -\hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}$ The adjacent sides \overrightarrow{AB} and \overrightarrow{BC} of the given rectangle are given as. $\overrightarrow{AB} = (1+1)\hat{i} + (\frac{1}{2} - \frac{1}{2})\hat{j} + (4-4)\hat{k} = 2\hat{i}$ $\overrightarrow{BC} = (1-1)\hat{i} + (-\frac{1}{2}-\frac{1}{2})\hat{j} + (4-4)\hat{k} = -\hat{j}$ $\therefore \overrightarrow{AB} \times \overrightarrow{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} = \hat{k} (-2) = -2\hat{k}$ $\Rightarrow \left| \overrightarrow{AB} \times \overrightarrow{BC} \right| = 2$

 $-\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}, \ \hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}, \ \hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}$ and $-\hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}$

(B) 1

(D) 4

We know that, the area of parallelogram whose adjacent sides are \vec{a} and \vec{b} is $|\vec{a} \times \vec{b}|$.

Thus, the area of the given rectangle is $|\overrightarrow{AB} \times \overrightarrow{BC}| = 2$ square units. So, the correct answer is C.

Miscellaneous Exercise

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1. Write down a unit vector in XY-plane, making an angle of 30° with the positive direction of *x*axis.

Solution:

If \vec{r} is a unit vector in the XY-plane, then $\vec{r} = \cos\theta \hat{i} + \sin\theta \hat{j}$.

Here, θ is the angle made by the unit vector with the positive direction of the *x*-axis. Hence, for $\theta = 30^{\circ}$ we have:

$$\vec{r} = \cos 30^{\circ} \hat{i} + \sin 30^{\circ} \hat{j} = \frac{\sqrt{3}}{2} \hat{i} + \frac{1}{2} \hat{j}$$

$$\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$$

Therefore, the required unit vector is 2

2. Find the scalar components and magnitude of the vector joining the points P (x₁, y₁, z₁) and Q (x₂, y₂, z₂). Solution:

The vector joining the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ can be found out by:

 \overrightarrow{PQ} = Position vector of Q – Position vector of P

$$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$\left| \overrightarrow{PQ} \right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Therefore, the scalar components and the magnitude of the vector joining the given points are respectively

$$\{(x_2 - x_1), (y_2 - y_1), (z_2 - z_1)\}$$
 and $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

3. A girl walks 4 km towards west, then she walks 3 km in a direction 30° east of north and stops. Determine the girl's displacement from her initial point of departure. Solution:

Let O and B be the initial and final positions of the girl respectively. Then, the girl's position can be shown as:



$$\overrightarrow{OA} = -4\hat{i}$$

$$\overrightarrow{AB} = \hat{i} |\overrightarrow{AB}| \cos 60^\circ + \hat{j} |\overrightarrow{AB}| \sin 60^\circ$$

$$= \hat{i} \cdot 3 \times \frac{1}{2} + \hat{j} \cdot 3 \times \frac{\sqrt{3}}{2}$$

$$= \frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$$

By the Triangle law of vector addition, we have $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$

$$= \left(-4\hat{i}\right) + \left(\frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}\right)$$
$$= \left(-4 + \frac{3}{2}\hat{j}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}\right)$$
$$= \left(\frac{-8+3}{2}\hat{j}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}\right)$$
$$= \frac{-5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$$

Therefore, the girl's displacement from her initial point of departure is





4. If $\vec{a} = \vec{b} + \vec{c}$, then is it true that $|\vec{a}| = |\vec{b}| + |\vec{c}|$? Justify your answer. Solution:

In $\triangle ABC$, let $\overrightarrow{CB} = \vec{a}$, $\overrightarrow{CA} = \vec{b}$, and $\overrightarrow{AB} = \vec{c}$ (as shown in the following figure).

So, by the Triangle law of vector addition, we have $\vec{a} = \vec{b} + \vec{c}$.

And, we know that $|\vec{a}|$, $|\vec{b}|$, and $|\vec{c}|$ represent the sides of ΔABC . Also, it is known that the sum of the lengths of any two sides of a triangle is greater than the third side.

 $\therefore \left| \vec{a} \right| < \left| \vec{b} \right| + \left| \vec{c} \right|$

Therefore, it is not true that $\left|\vec{a}\right| = \left|\vec{b}\right| + \left|\vec{c}\right|$

5. Find the value of x for which $x(\hat{i}+\hat{j}+\hat{k})$ is a unit vector. Solution:

Given
$$x(\hat{i} + \hat{j} + \hat{k})$$
 is a unit vector.
So, $|x(\hat{i} + \hat{j} + \hat{k})| = 1$
Now,
 $|x(\hat{i} + \hat{j} + \hat{k})| = 1$
 $\sqrt{x^2 + x^2 + x^2} = 1$
 $\sqrt{3x^2} = 1$
 $\sqrt{3x^2} = 1$
 $x = \pm \frac{1}{\sqrt{3}}$
Therefore, the required value of x is $\pm \frac{1}{\sqrt{3}}$

6. Find a vector of magnitude 5 units, and parallel to the resultant of the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$.



EDUGROSS

WISDOMISING KNOWLEDGE

Given vectors, $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$ Let \vec{c} be the resultant of \vec{a} and \vec{b} . Then, $\vec{c} = \vec{a} + \vec{b} = (2+1)\hat{i} + (3-2)\hat{j} + (-1+1)\hat{k} = 3\hat{i} + \hat{j}$ $|\vec{c}| = \sqrt{3^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$ $\therefore \hat{c} = \frac{\vec{c}}{|\vec{c}|} = \frac{(3\hat{i} + \hat{j})}{\sqrt{10}}$

Therefore, the vector of magnitude 5 units and parallel to the resultant of vectors \vec{a} and \vec{b} is

$$\pm 5 \cdot \hat{c} = \pm 5 \cdot \frac{1}{\sqrt{10}} \left(3\hat{i} + \hat{j} \right) = \pm \frac{3\sqrt{10}\hat{i}}{2} \pm \frac{\sqrt{10}}{2} \hat{j}.$$

$$= \frac{1}{10} \hat{a} + \hat{j} + \hat{k}, \quad \vec{b} = 2\hat{i} - \hat{j} + 3\hat{k} \text{ and } \vec{c} = \hat{i} - 2\hat{j} + \hat{k}$$

7. If
$$a = i + j + k$$
, $b = 2i - j + 3k$ and $c = i - 2j - 2\vec{a} - \vec{b} + 3\vec{c}$

, find a unit vector parallel to the vector

Solution:

Given,

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}, \ \vec{b} = 2\hat{i} - \hat{j} + 3\hat{k} \text{ and } \vec{c} = \hat{i} - 2\hat{j} + \hat{k}$$

 $2\vec{a} - \vec{b} + 3\vec{c} = 2(\hat{i} + \hat{j} + \hat{k}) - (2\hat{i} - \hat{j} + 3\hat{k}) + 3(\hat{i} - 2\hat{j} + \hat{k})$
 $= 2\hat{i} + 2\hat{j} + 2\hat{k} - 2\hat{i} + \hat{j} - 3\hat{k} + 3\hat{i} - 6\hat{j} + 3\hat{k}$
 $= 3\hat{i} - 3\hat{j} + 2\hat{k}$
 $\left|2\vec{a} - \vec{b} + 3\vec{c}\right| = \sqrt{3^2 + (-3)^2 + 2^2} = \sqrt{9 + 9 + 4} = \sqrt{22}$
Therefore, the unit vector along $2\vec{a} - \vec{b} + 3\vec{c}$ is
 $\frac{2\vec{a} - \vec{b} + 3\vec{c}}{|2\vec{a} - \vec{b} + 3\vec{c}|} = \frac{3\hat{i} - 3\hat{j} + 2\hat{k}}{\sqrt{22}} = \frac{3}{\sqrt{22}}\hat{i} - \frac{3}{\sqrt{22}}\hat{j} + \frac{2}{\sqrt{22}}\hat{k}.$

8. Show that the points A (1, -2, -8), B (5, 0, -2) and C (11, 3, 7) are collinear, and find the ratio in which B divides AC.

Given points are: A
$$(1, -2, -8)$$
, B $(5, 0, -2)$, and C $(11, 3, 7)$.
 $\therefore \overrightarrow{AB} = (5-1)\hat{i} + (0+2)\hat{j} + (-2+8)\hat{k} = 4\hat{i} + 2\hat{j} + 6\hat{k}$
 $\overrightarrow{BC} = (11-5)\hat{i} + (3-0)\hat{j} + (7+2)\hat{k} = 6\hat{i} + 3\hat{j} + 9\hat{k}$
 $\overrightarrow{AC} = (11-1)\hat{i} + (3+2)\hat{j} + (7+8)\hat{k} = 10\hat{i} + 5\hat{j} + 15\hat{k}$
 $\left|\overrightarrow{AB}\right| = \sqrt{4^2 + 2^2 + 6^2} = \sqrt{16 + 4 + 36} = \sqrt{56} = 2\sqrt{14}$
 $\left|\overrightarrow{BC}\right| = \sqrt{6^2 + 3^2 + 9^2} = \sqrt{36 + 9 + 81} = \sqrt{126} = 3\sqrt{14}$
 $\left|\overrightarrow{AC}\right| = \sqrt{10^2 + 5^2 + 15^2} = \sqrt{100 + 25 + 225} = \sqrt{350} = 5\sqrt{14}$
 $\therefore \left|\overrightarrow{AC}\right| = \left|\overrightarrow{AB}\right| + \left|\overrightarrow{BC}\right|$

Therefore, the given points A, B, and C are collinear. Now, let point B divide AC in the ratio λ :1. So, we have:

$$\overline{OB} = \frac{\lambda OC + OA}{(\lambda + 1)}$$

$$5\hat{i} - 2\hat{k} = \frac{\lambda \left(11\hat{i} + 3\hat{j} + 7\hat{k}\right) + \left(\hat{i} - 2\hat{j} - 8\hat{k}\right)}{\lambda + 1}$$

$$(\lambda + 1) \left(5\hat{i} - 2\hat{k}\right) = 11\lambda\hat{i} + 3\lambda\hat{j} + 7\lambda\hat{k} + \hat{i} - 2\hat{j} - 8\hat{k}$$

$$5(\lambda + 1)\hat{i} - 2(\lambda + 1)\hat{k} = (11\lambda + 1)\hat{i} + (3\lambda - 2)\hat{j} + (7\lambda - 8)\hat{k}$$
On equating the corresponding components, we have:

 $5(\lambda + 1) = 11\lambda + 1$ $5\lambda + 5 = 11\lambda + 1$ $6\lambda = 4$ $\lambda = \frac{4}{6} = \frac{2}{3}$

Therefore, point B divides AC in the ratio 2:3.

9. Find the position vector of a point R which divides the line joining two points P and Q whose

position vectors are $(2\vec{a}+\vec{b})$ and $(\vec{a}-3\vec{b})$ externally in the ratio 1: 2. Also, show that P is the mid point of the line segment RQ. Solution:

Given,
$$\overrightarrow{OP} = 2\vec{a} + \vec{b}, \ \overrightarrow{OQ} = \vec{a} - 3\vec{b}$$

Also, given that point R divides a line segment joining two points P and Q externally in the ratio 1: 2. So, on using the section formula, we have

$$\overrightarrow{\text{OR}} = \frac{2(2\vec{a} + \vec{b}) - (\vec{a} - 3\vec{b})}{2 - 1} = \frac{4\vec{a} + 2\vec{b} - \vec{a} + 3\vec{b}}{1} = 3\vec{a} + 5\vec{b}$$

Hence, the position vector of point R is $3\vec{a} + 5\vec{b}$. Now,

Position vector of the mid-point of RQ = $\frac{\overrightarrow{OQ} + \overrightarrow{OR}}{2}$

$\mathbf{RQ} = \frac{2}{2}$ $= \frac{\left(\vec{a} - 3\vec{b}\right) + \left(3\vec{a} + 5\vec{b}\right)}{2}$ $= 2\vec{a} + \vec{b}$ $= \overrightarrow{OP}$

Therefore, P is the mid-point of the line segment RQ.

10. The two adjacent sides of a parallelogram are $2\hat{i}-4\hat{j}+5\hat{k}$ and $\hat{i}-2\hat{j}-3\hat{k}$. Find the unit vector parallel to its diagonal. Also, find its area. Solution:

Adjacent sides of a parallelogram are given as: $\vec{a} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} - 3\hat{k}$ We know that, the diagonal of a parallelogram is given by $\vec{a} + \vec{b}$. $\vec{a} + \vec{b} = (2+1)\hat{i} + (-4-2)\hat{j} + (5-3)\hat{k} = 3\hat{i} - 6\hat{j} + 2\hat{k}$

Hence, the unit vector parallel to the diagonal is

$$\frac{\vec{a}+\vec{b}}{\left|\vec{a}+\vec{b}\right|} = \frac{3\hat{i}-6\hat{j}+2\hat{k}}{\sqrt{3^2+(-6)^2+2^2}} = \frac{3\hat{i}-6\hat{j}+2\hat{k}}{\sqrt{9+36+4}} = \frac{3\hat{i}-6\hat{j}+2\hat{k}}{7} = \frac{3}{7}\hat{i}-\frac{6}{7}\hat{j}+\frac{2}{7}\hat{k}.$$



So, the area of parallelogram ABCD = $\begin{vmatrix} \vec{a} \times \vec{b} \end{vmatrix}$ $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 5 \end{vmatrix}$

$$\begin{vmatrix} 1 & -2 & -3 \end{vmatrix}$$

= $\hat{i} (12+10) - \hat{j} (-6-5) + \hat{k} (-4+4)$
= $22\hat{i} + 11\hat{j}$
= $11(2\hat{i} + \hat{j})$
 $\therefore |\vec{a} \times \vec{b}| = 11\sqrt{2^2 + 1^2} = 11\sqrt{5}$

Therefore, the area of the parallelogram is $11\sqrt{5}$ square units.

11. Show that the direction cosines of a vector equally inclined to the axes OX, OY and OZ are

 $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

Solution:

Let's assume a vector to be equally inclined to axes OX, OY, and OZ at angle α . Then, the direction cosines of the vector are $\cos \alpha$, $\cos \alpha$, and $\cos \alpha$. Now, we know that

 $\cos^{2} \alpha + \cos^{2} \alpha + \cos^{2} \alpha = 1$ $3\cos^{2} \alpha = 1$ $\cos \alpha = \frac{1}{\sqrt{3}}$

Therefore, the direction cosines of the vector which are equally inclined to the axes are

 $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

$$\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}, \ \vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$$
 and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$
12. Let . Find a vector \vec{d} which is perpendicular to both Solution:

Let $\vec{d} = d_1\hat{i} + d_2\hat{j} + d_3\hat{k}$. As \vec{d} is perpendicular to both \vec{a} and \vec{b} , we have: $\vec{d} \cdot \vec{a} = 0$ $d_1 + 4d_2 + 2d_3 = 0$...(i)

And, $\vec{d} \cdot \vec{b} = 0$ $3d_1 - 2d_2 + 7d_3 = 0$...(ii) Also, given that: $\vec{c} \cdot \vec{d} = 15$ $2d_1 - d_2 + 4d_3 = 15$...(iii) On solving (i), (ii), and (iii), we obtain $d_1 = \frac{160}{3}, d_2 = -\frac{5}{3}$ and $d_3 = -\frac{70}{3}$ $\therefore \vec{d} = \frac{160}{3}\hat{i} - \frac{5}{3}\hat{j} - \frac{70}{3}\hat{k} = \frac{1}{3}(160\hat{i} - 5\hat{j} - 70\hat{k})$ Therefore the required vector is $\frac{1}{3}(160\hat{i} - 5\hat{j} - 70\hat{k})$

Therefore, the required vector is $\frac{1}{3} \left(160\hat{i} - 5\hat{j} - 70\hat{k} \right)$

13. The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors and $\hat{i} + \hat{j} + \hat{k}$ is equal to one. Find the value of λ .

Sum of the given vectors is given by, $(2\hat{i}+4\hat{j}-5\hat{k})+(\lambda\hat{i}+2\hat{j}+3\hat{k}) = (2+\lambda)\hat{i}+6\hat{j}-2\hat{k}$ Hence, unit vector along $(2\hat{i}+4\hat{j}-5\hat{k})+(\lambda\hat{i}+2\hat{j}+3\hat{k})$ is given as: $\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(2+\lambda)^2}+6^2+(-2)^2} = \frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{4+4\lambda+\lambda^2}+36+4} = \frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{\lambda^2}+4\lambda+44}$ Scalar product of $(\hat{i}+\hat{j}+\hat{k})$ with this unit vector is 1. $(\hat{i}+\hat{j}+\hat{k})\cdot\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{\lambda^2}+4\lambda+44} = 1$ $\frac{(2+\lambda)+6-2}{\sqrt{\lambda^2}+4\lambda+44} = 1$ $\frac{(2+\lambda)+6-2}{\sqrt{\lambda^2}+4\lambda+44} = \lambda+6$ $\lambda^2+4\lambda+44 = (\lambda+6)^2$ $\lambda^2+4\lambda+44 = (\lambda+6)^2$ $\lambda=1$ Therefore, the value of A is 1.

- 14. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a}, \vec{b}, \vec{c}$
 - $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to \vec{a}, \vec{b} and \vec{c} . Solution:



$$\begin{aligned} |\vec{a} \cdot \vec{c} \cdot \vec{c}||^{e_1} & |\vec{a} \cdot \vec{c} \cdot \vec{c}||^{e_1} \\ &= \frac{|\vec{a}|^2}{|\vec{a} + \vec{b} + \vec{c}| |\vec{a}|} & [\vec{b} \cdot \vec{a} = \vec{c} \cdot \vec{a} = 0] \\ &= \frac{|\vec{a}|}{|\vec{a} + \vec{b} + \vec{c}|} \\ \cos\theta_2 &= \frac{(\vec{a} + \vec{b} + \vec{c}) \cdot \vec{b}}{|\vec{a} + \vec{b} + \vec{c}| \cdot |\vec{b}|} = \frac{\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{b} + \vec{c} \cdot \vec{b}}{|\vec{a} + \vec{b} + \vec{c}| \cdot |\vec{b}|} \\ &= \frac{|\vec{b}|^2}{|\vec{a} + \vec{b} + \vec{c}| \cdot |\vec{b}|} & [\vec{a} \cdot \vec{b} = \vec{c} \cdot \vec{b} = 0] \\ &= \frac{|\vec{b}|}{|\vec{a} + \vec{b} + \vec{c}|} \\ \cos\theta_3 &= \frac{(\vec{a} + \vec{b} + \vec{c}) \cdot \vec{c}}{|\vec{a} + \vec{b} + \vec{c}| |\vec{c}|} = \frac{\vec{a} \cdot \vec{c} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{c}}{|\vec{a} + \vec{b} + \vec{c}| |\vec{c}|} \\ &= \frac{|\vec{c}|^2}{|\vec{a} + \vec{b} + \vec{c}| |\vec{c}|} & [\vec{a} \cdot \vec{c} = \vec{b} \cdot \vec{c} = 0] \\ &= \frac{|\vec{c}|}{|\vec{a} + \vec{b} + \vec{c}|} \\ \text{Now, as} |\vec{a}| = |\vec{b}| = |\vec{c}|, \ \cos\theta_1 = \cos\theta_2 = \cos\theta_3. \end{aligned}$$

$$\therefore \theta_1 = \theta_2 = \theta_3$$

Therefore, the vector $(\vec{a} + \vec{b} + \vec{c})$ is equally inclined to \vec{a}, \vec{b} , and \vec{c} .

15. Prove that $(\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) = |\vec{a}|^2 + |\vec{b}|^2$, if and only if \vec{a} , \vec{b} are perpendicular, given $\vec{a} \neq \vec{0}$, $\vec{b} \neq \vec{0}$. Solution:

Required to prove: $\left(\vec{a}+\vec{b}\right)$. $\left(\vec{a}+\vec{b}\right) = \left|\vec{a}\right|^2 + \left|\vec{b}\right|^2$ $\vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} + \vec{b} \cdot \vec{b} = \left| \vec{a} \right|^2 + \left| \vec{b} \right|^2$ [Distributivity of scalar products over addition] $\left|\vec{a}\right|^{2} + 2\vec{a}\cdot\vec{b} + \left|\vec{b}\right|^{2} = \left|\vec{a}\right|^{2} + \left|\vec{b}\right|^{2}$ $\begin{bmatrix} \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a} & \text{(Scalar product is commutative)} \end{bmatrix}$ $2\vec{a}\cdot\vec{b}=0$ $\vec{a} \cdot \vec{b} = 0$ $\begin{bmatrix} \vec{a} \neq \vec{0}, \ \vec{b} \neq \vec{0} \ \text{(Given)} \end{bmatrix}$ Therefore, \vec{a} and \vec{b} are perpendicular. 16. If θ is the angle between two vectors \vec{a} and \vec{b} , then $\vec{a}.\vec{b} \ge 0$ $0 < \theta < \frac{\pi}{2}$ $0 \le \theta \le \frac{\pi}{2}$ (A) (B) (C) $0 < \theta < \pi$ **(D)** $0 \le \theta \le \pi$ only when Solution: Let's assume θ to be the angle between two vectors a and b. Then, without loss of generality, \vec{a} and \vec{b} are non-zero vectors so that $|\vec{a}|$ and $|\vec{b}|$ are positive We also know, $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$ So, $\vec{a} \cdot \vec{b} \ge 0$ $|\vec{a}| |\vec{b}| \cos \theta \ge 0$ $\left[\left| \vec{a} \right| \text{ and } \left| \vec{b} \right| \text{ are positive} \right]$ $\cos\theta \ge 0$ $0 \le \theta \le \frac{\pi}{2}$ Therefore, $\vec{a}.\vec{b} \ge 0$ when $0 \le \theta \le \frac{\pi}{2}$ The correct answer is B. 17. Let \vec{a} and \vec{b} be two unit vectors and θ is the angle between them. Then $\vec{a} + \vec{b}$ is a unit vector $\theta = \frac{\pi}{4}$ (B) $\theta = \frac{\pi}{3}$ (C) $\theta = \frac{\pi}{2}$ $\theta = \frac{2\pi}{3}$ if (A) (D)

Let \vec{a} and \vec{b} be two unit vectors and θ be the angle between them. Then, $\left| \vec{a} \right| = \left| \vec{b} \right| = 1$. Now, $\vec{a} + \vec{b}$ is a unit vector if $|\vec{a} + \vec{b}| = 1$. $\left| \vec{a} + \vec{b} \right| = 1$ $\left(\vec{a}+\vec{b}\right)^2 = 1$ $(\vec{a}+\vec{b}).(\vec{a}+\vec{b})=1$ $\vec{a}.\vec{a}+\vec{a}.\vec{b}+\vec{b}.\vec{a}+\vec{b}.\vec{b}=1$ $\left|\vec{a}\right|^{2} + 2\vec{a}.\vec{b} + \left|\vec{b}\right|^{2} = 1$ $1^{2} + 2\left|\vec{a}\right| \left|\vec{b}\right| \cos \theta + 1^{2} = 1$ $1+2.1.1\cos\theta+1=1$ $\cos\theta = -\frac{1}{2}$ $\theta = \frac{2\pi}{3}$ Therefore, $\vec{a} + \vec{b}$ is a unit vector if $\theta = \frac{2\pi}{3}$. 18. The value of $\hat{i}.(\hat{j}\times\hat{k})+\hat{j}.(\hat{i}\times\hat{k})+\hat{k}.(\hat{i}\times\hat{j})$ is (A) 0 **(B)**-1 (C) 1 **(D)** 3 Solution: Given, $\hat{i}.\left(\hat{j}\times\hat{k}\right)+\hat{j}.\left(\hat{i}\times\hat{k}\right)+\hat{k}.\left(\hat{i}\times\hat{j}\right)$ $=\hat{i}\cdot\hat{i}+\hat{j}\cdot\left(-\hat{j}\right)+\hat{k}\cdot\hat{k}$ $=1-\hat{j}\cdot\hat{j}+1$ =1-1+1= 1The correct answer is C.

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WISDOMISING KNOWLEDGE
           19. If \theta is the angle between any two vectors \vec{a} and \vec{b}, then |\vec{a}.\vec{b}| = |\vec{a}\times\vec{b}|
                                           (B) \frac{\pi}{4}
                                                          (C) \frac{\pi}{2}
            (A) 0
                                                                                              (D) π
                                                                                                                                     when \theta is equal to
                                                                                                                                                    Solution:
            Let \theta be the angle between two vectors \vec{a} and \vec{b}.
            Then, without loss of generality, \vec{a} and \vec{b} are non-zero vectors, so that |\vec{a}| and |\vec{b}| are positive.
             \left| \vec{a} \cdot \vec{b} \right| = \left| \vec{a} \times \vec{b} \right|
             |\vec{a}||\vec{b}|\cos\theta = |\vec{a}||\vec{b}|\sin\theta
                                     \left| \vec{a} \right| and \left| \vec{b} \right| are positive
             \cos\theta = \sin\theta
             \tan \theta = 1
             \theta = \frac{\pi}{4}
             Thus, \left|\vec{a}.\vec{b}\right| = \left|\vec{a}\times\vec{b}\right| when \theta is equal to \frac{\pi}{4}
             So, the correct answer is B.
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